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Probability*

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THE subject of probability reminds one of that Australian curiosity enjoying the orotund and circus-stirring name of "ornithorhyncus Paradoxus or duck-billed Platypus"—is it fish or fowl or beast?" Like the sacred cod of Boston this strange animal lives in the water; like the royal swans of his Britannic majesty, it has a flat bill, web-feet, and lays white shell-encased eggs; like the tiger of Kipling's *Jungle Tales* it has soft fur and suckles its young. And beside all this the male has spurs like a rooster. In a somewhat analogous way, probability, like the quadratic equation, or the removal of parentheses, is a subject in textbooks on algebra. Along with partial differential equations it is one of the advanced topics in mathematical physics. In connection with statistics it comes under education, economics, psychology, biology, or mathematics. It is studied in the theory of manufacturing practice, and in describing the operation of genetic inheritance. It proves absorbing to the dissolute gambler, while the philosopher treats it as an abstruse item of theoretical logic. It is invoked in casual comments about the weather, explained in treatises on practical surveying, and constitutes one of the first principles in the rational discussion of old age pensions, and of business cyles. Is it not strange that to explain how heat causes water to boil, one may well consider first the effect of tossing coins? Where else in algebra is any such economic prudence as that of life insurance brought home to the student, and where else is he asked to contemplate games of dice, gambling at Monte Carlo, and luck in drawing cards for poker?

One authority says that probability is a mental attitude, a second that it is an arithmetic ratio, another that it is the expected outcome in a long series of trials, a fourth that it is a modal quality of propositional forms. Surely it out-ornithorhyncuses the Ornithorhyncus Paradoxus, in the diversity of its uncles, and its cousins and its aunts. There is however this marked difference—*the Ornithorhyncus is almost extinct.*

Probability is a subject more than usually rich in the following respects.

1. In questions of a philosophical and logical nature. The definition of probability, the paradoxes such as that known by the name of St. Petersburg, the validity of Bayes' formula, the basis for the Euler-Laplace normal law, the nature of ge-

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ometric probability, the scope of the law of small numbers, the relation of probability to deterministic theories of causality—these to mention but a few are matters still open to argument and to clash of opinion among authorities.

2. In problem material. In few branches are there so many easily stated but independently baffling exercises for the clever student to solve, and for the mere imitator to flounder among. Change but a word or two and one has a new problem demanding a new twist of ideas.

3. In elegant proofs. Even in the finite case of a binomial distribution, the theorems follow by unexpectedly powerful methods of elementary analysis, and delight every true mathematician.

4. In variety of applications, as already hinted at.

The practical considerations of a limited time, a large audience, little or no blackboard space and hot weather suggest that I abstain from leading you through any succession of problem solutions, any sequence of deductive and demonstrative stages and any elaboration of the refinements in applications. I must be content to talk about the subject and to suggest if I may rather than to point out wide vistas that lie beyond, although this procedure cannot do justice to the topic.

As to books on the subject, one may say that nearly all texts on statistics contain one or more chapters on probability. More extensive treatises devoted entirely to probability are, to mention only some recent outstanding works the following twenty-five.

J. L. F. Bertrand, Calcul des probabilités. 2nd Ed., 1907.

Émile Borel, Some seven works, including Le hasard. 2nd Ed., 1914, and Éléments de la théorie des probabilités. 2nd Ed., 1910.

W. Burnside, Theory of probability. 1928

J. L. Coolidge, An introduction to mathematical probability. 1925.

E. Czuber, Wahrscheinlichkeitsrechnung. 1914. 2 vols.

L. G. Du Pasquier, Le calcul des probabilités. 1926.

Bruno de Finetti, Probabilismo. 1931.

Arne Fisher, The mathematical theory of probability. 2nd Ed., 1922.

T. C. Fry, Probability and its engineering uses. 1928.

Marsh Hopkins, Choice and chance. 1923.

E. Kamke, Einführung in die Wahrscheinlichkeitstheorie. 1932.

J. M. Keynes, A treatise on probability. 1921.

O. H. J. Knopf, Wahrscheinlichkeitsrechnung. 1923. 2 vols.

A. N. Kolmogorov, Grundbegriffe der Wahrscheinlichkeitsrechnung. 1933.

J. von Kries, Die principien der Wahrscheinlichkeitsrechnung. 1927. 2 vols.

Paul Lévy, Calcul des probabilités. 1925.

Paul Mansion, Calcul des probabilités. 1916.

A. A. Markov, Wahrscheinlichkeitsrechnung. 1912.

Otto Meissner, Wahrscheinlichkeitsrechnung. 1912.

R. von Mises, Wahrscheinlichkeitsrechnung. 1931.

R. F. B. Montessus de Ballore, Leçons élémentaires sur le calcul des probabilités. 1908.

Hans Reichenbach, Wahrscheinlichkeitsrechnung. 1931.

A. Sainte-Laguë, Probabilités et morphologie. 1932.

F. M. Urban, Grundlagen der Wahrscheinlichkeitsrechnung. 1923.

P. van Deuren, Leçons sur le calcul de probabilités. 1935. 2 vols.

I shall have little to say concerning elementary probability as a topic in secondary school mathematics. It has its good points and also its obvious disadvantages. To mention some items in its favor let me remark:

1. It awakens instant interest and astonishes the pupils by introducing formulas into unexpected angles of daily life.

2. It develops naturally out of the theory of permutations and combinations, which in turn serves in making clear the binomial theorem.

3. In connection with permutations it cultivates an educated common sense by drawing out of apparently harmless situations, numbers of such stupendous magnitude, that only confidence in reason and the experience with analogous but even simpler cases, serve to make these wholly credible.

4. It prepares the way for statistical

discussions of immediate application to important and widely diverse fields of human activity.

5. It yields multitudes of easily stated but somewhat baffling problems whose conquest the good students find exhilarating.

On the other side of the ledger stand the following perhaps fatal entries:

1. It is a subject which cannot be mastered without original thinking, and in which the faithful plodder with reliable memory, and with technical skill in formal operations may be completely confounded.

2. It aids little in clinching earlier topics and is not called upon in the prolonged struggles through trigonometry or analytic geometry.

3. Unlike most mathematical topics, authorities in the theory of probability disagree about the definitions, rules, values, and even about the answers to numerical problems.

Let me explain in a little more detail as to the nature of probability.

1. Common sense and tradition seem to suggest that probability as an arithmetic ratio resides in the event, or outcome of trials. The uncompleted experiment seems to possess some stated probability of success. The tossed coin is said to have the probability one-half of turning up heads. The galloping domino (in the plural, dice, to you) has the probability one-sixth of showing a given number of spots (from one to six), and similarly for packs of cards and the like. There are difficulties with this simple objective view. What happens when the test is finished? What makes the probability suddenly change to unity or to zero? And again, does it change for the person who has not yet learned the facts? Let me be specific. Suppose I spin a coin. I spin it fairly and the coin is an honest one. You tell me that the probability is one-half that it will show a head. Of course this is before the coin falls on its side. When the spin is over, and we look at the coin as it lies on the table, the feature of chance has been eliminated. The probability if we still wish to use that term, is definitely unity or zero, according as it does or does not show a head. But suppose when I spin the coin, it rolls under the table and falls flat on the floor, I can see it dimly lying there but cannot make out as yet which side is uppermost. Is the probability that it lies with the head showing still one-half? Of course the result is now fully determined, but I do not know the answer. It seems best to admit that the probability is still one-half. For after all, even when I first spun the coin, the forces determining the result were already in operation. In a deterministic philosophy the result is settled but unknown and practically unknowable. Something definitely happens to the probability, when I look at the coin. And since the result of the toss has not itself altered, the probability cannot lie wholly in the event. In literary parlance, I may bend my gaze or glue my eyes to the floor, but mere looking, at least of the sort that I could do, would not convert the filthy lucre and cause it to turn over a new leaf by a complete face-about. Any view has difficulties. Suppose I asked you, what day of the week was July 4, 1776. Very likely you do not know and conclude that any day of the week is equally likely, perhaps forgetting the sanctity of the Sabbath—and so the probability of its being Thursday, you say is one-seventh. Of course you may not know, but here you have plenty of evidence at hand and can find out for yourself. I should say that the matter is either a certainty or an impossibility, and probability at least of causes, is inoperative. Let us proceed to find out. By 1776 the present Gregorian calendar was in force in these colonies. By subtraction $1936-1776=160$. Of these 160 years every fourth was a leap year, save for the two years 1800 and 1900 which were ordinary years. Instead of adding 40 days for the leap years, we add only 38 days. To compare July 4, 1776 with July 4, 1936, we wish to find the remainder on dividing by 7, of the expres-

sion $(160\times 365)+38$. Since 365 divided by 7 leaves a remainder 1, it suffices to divide $160+38$ by 7. The remainder is 2. Hence this year July 4, coming on Saturday, came two days later in the week, than it did in 1776. Thus July 4, 1776 was certainly Thursday.

Let us turn to another view.

2. Probability is said to measure ignorance. Probability then depends upon the person's state of knowledge, and may be different for different people—certainly might be different for the analytic-minded detective from what it is for the stupid or casual onlooker. This seems to make probability hardly mathematical, and scarcely applicable to objective occurrences. This view can be exploited to quite ridiculous extremes and by illogical methods. One has argued as follows. Suppose, when I am in the cellar looking after the furnace, I hear a bell ring. I do not know whether it is my own door bell or the neighbor's. Hence one might say the probability of its being my door bell is one-half. But if it be my door bell, and my son does not answer it, the bell-ringer or button-pusher, may be a beggar. Hence the probability that the door bell is mine, that my boy does not answer it and that the person be not a beggar is only $\frac{1}{8}$. Again, if it is not a beggar it may be a book-agent, I do not know. Hence the probability that it is my door bell, that my boy does not answer it, that it is not a beggar nor a book-agent, is $\frac{1}{16}$. But if not a book-agent, I do not know whether it might be a bill collector, whom I do not desire to entertain. Hence the probability that it is my door bell, that my boy is out, that it is neither a beggar, book-agent, or bill collector is 1/32. Proceeding in this quite illegitimate manner one soon arrives at the conclusion, that the probability of there being any sense in my hurrying upstairs, wiping off the ashes and going to the door, is less than 10^{-57}, so of course I should ignore the sound.

We might go further and consider.

3. Probability is sometimes called the measure of expectation, the mental basis for the odds which one is willing to offer on a future outcome. This view has much to commend it at first sight. People who bet suppose that probability favors the chances they are taking. When I ask what is the probability of a European war within ten years, or what is the chance that my infant nephew will become President of the United States, or that little Maggie will be a musical genius, or that I will give in and take the children to the next circus, or that 4 year old Johnnie will get the mumps the first year he goes to school, it is doubtless some such sense of probability that we have in mind. To the timorous and the moralistic, the miser and the pauper, who will not bet whatever the odds, probability seems then to reside at a constant low tide level of zero. Anger, cheerful news, indigestion, friendly persuasion, despair, all tend to change the odds—even in a simple social game of contract bridge, so that probability then becomes a study of emotional states and no fit subject for the mathematician. But there are other views.

4. Probability has been claimed to be an average in the long run of the ratio of favorable outcomes to total number of trials. This surely has much to commend it. However much we know about coins and dice and packs of cards, there are many things which we cannot learn about on any such a priori grounds as govern these familiar cases. For example if we wish to know the probable number of feathers in the tail of a pouter pigeon, or the probable range of a gun, or the probable number of minute steaks called for at Saturday noon in a downtown restaurant, or the number of umbrellas left in the trains during the Easter holidays, or the number of pupils who will fail to pass their mathematics tests, and so forth, we must resort to the thoroughly commonplace procedure of counting and classifying. But when we recall that probability thus defined is really only a limit approached as the number of trials becomes

infinitely great, we stop to ponder. How can any finite sequence of trials tell us about this limit? Can a thousand, even a million experiments assure us of the really eventual trend? Far from being confident of our powers in evaluating such a limit, what reason have we to suppose that such a limit exists? Indeed probability seems to become a goal whose very existence is only a matter of postulate or fancy, and whose numerical value remains shrouded in eternal clouds like some mathematical Mt. Everest, or perhaps better, like the ropes that the Indian fakirs are said to unroll toward heaven.

5. There is a view which denies that probability is even related to individual events, and reserves its application to extensive regular classes of events.

Let me illustrate. One may say that the probability of a man aged 40 living 30 more years is approximately one-half. But in saying this we should never make the application to any one given person of 40 years of age. There are too many additional factors to render the statement valid. One merely means that of the class of all men, now aged 40, and contemplated by the given mortality table, about one-half may be expected to be alive at 70 years of age. For unique events, probability becomes inappropriate, such as for the occurrence of war, or election of a given person. Indeed a given individual may have different probable lengths of life according to what classification he is being subjected to.

But all these descriptions of the nature of probability seem open to devastating criticisms. A view much favored in many quarters today is that probability characterizes propositions and not events. Facts, even future facts seem bare and solid and not decked out with the soft diaphanous veils of probability. And so I mention

6. Probability is said to be an objective quality of propositions, but a function of the evidence. According to this view one should say that such and such a proposition has a stated probability upon the basis of such and such evidence. If fresh facts bearing upon the situation are learned, then although the original proposition may be the same, its probability has changed. This view restores to probability its objective character. The evidence should be weighed and analyzed impersonally and freed from all emotional bias. Since however, the weighing of evidence is hardly mathematical, this view would seem at first to remove the subject from mathematical investigation. At best this view tells only part of the story, because although evidence will change the situation it need not change the probability. It may add assurance that one estimate of probability is correct.

Then again, just what evidence have we concerning coins and dice and cards? If the man in the south who is recording the outcome of tossing a coin 50,000 times should decide that heads come up a little more often than tails,—what then? This merely means that either he has not had a long enough run, or that he must learn to toss more fairly, or that the coin (or coins, for he may have worn out a few of them) are lop-sided. If the evidence does not support the theory then like a good Aristotelian, I say in this case, so much the worse for the experiment. In biology I will admit I do not trust Aristotle's reputed statement that flies have just four legs.

I do not desire to say much concerning traditional finite probability. You all will agree I suppose that on tossing a coin either side is equally likely to turn up—that is, for well-behaved coins that do not roll under the table, or slip down a crack, or just decide to face the future standing on edge like a hero. I suppose most of you agree with Bertrand that a tossed coin has neither conscience nor memory, and will be equally likely to show heads or tails, even if it should happen to be known that for thirty successive throws heads have turned up. For myself, I would demand inspecting such a coin to see whether like Janus it might not be two-

headed and be opening the door of opportunity to some rather shady enterprise. I should wish also to throw it myself the next time or two, to outwit the jinx or hoodoo or whatever it be. Suppose if after hands are dealt and bidding has been completed for a round of Contract, the hostess leaves the game to organize the commissary, calling upon some other table's dummy to play her hand. Her cards are laid in a neat pile face down upon the table. Doubtless most of you would argue that the probability of the top card of this pile being an ace is one in thirteen. But if you pause to consider the likelihood of her having arranged her cards by suits and in order within suits, the probability of the top card being an ace is obviously considerably increased.

Non-arithmetic considerations of such sorts repeatedly intrude to disturb the complacency of the ready reckoner of probabilities. Suppose that in the capital of a certain southern state, the wealth and chivalry of the old families gather for the wedding of the season's fairest debutante, the cherished daughter of the chief justice of the state's supreme court. The arithmetic fact that the state's population numbers seven negroes to every white person, in no way affects the likelihood as to the probable race of the bridegroom. In testing random groups of persons under hypotheses of probability, one must of course make allowance in the case of related persons, for children do indeed take after their parents, and parents do resemble their children, and yet even for a family of seven sisters all old maids, who would expect to find a single old maid in their direct ancestral line. Similarly strange as it may seem in view of the large number of deaths in infancy and childhood, not one of the ancestors of the present generation died in infancy.

We hear about problems inquiring as to how many committees of five can be organized from a group consisting of 8 Republicans and 5 Democrats. Certainly in one state I know, the answer is immediate—just one committee. Then there is the question as to what is the chance of finding four aces in the first four cards drawn from a pack. Who of you has not seen the sleight-of-hand artist draw cards out of the air, from behind an astonished neighbor's ear, or shyly peeking out from behind the edge of his trouser leg? He is perfectly capable of conjuring up the king of hearts out of a half full glass of water, and of discovering the queen of hearts clandestinely hiding with the jack of diamonds behind the parlor clock. If he told me that he would draw 8 aces in succession from a pack that I had just examined, I would not think of contradicting him. Then there is the question as to how many words can be formed with given four letter each used exactly once, say O,P,S,T. Now the following I admit are words, SPOT, STOP, POST, POTS, TOPS. But the required answer is supposed to be 24. Now do you call PTSO, or OTSP, words?

Of course there is no such thing as a negative probability, although more than one humble employee during the long drawn out depression, has figured that the probability of a raise in his salary should be fittingly expressed only by negative numbers. Nor again should one ever claim emphatic confidence in some event by declaring that its probability is at least a million per cent.

Now the problems in finite probability as usually given in algebra texts, although often baffling to the pupil are ordinarily rather simple affairs taking but a moment's time to one who has had acquaintance with such types of problems and who can think accurately along such lines. There is often more than one way of doing a problem, some ways being much shorter than others.

If a card is taken at random from an ordinary deck of playing cards, we assume that the probability of its being an ace is 1/13. What pupils usually find difficult to appreciate is that the "at random" continues to apply, no matter how we

distribute or shuffle the cards so long as they remain unknown. For example I might shuffle and deal the pack (as usual face down) into four "hands" of 13 cards each. Suppose each player draws one card at random from his hand still lying face down, looks at this card and places it in the middle of the table. An outsider who has seen none of the cards is to draw one of these. What is the probability that it is an ace? Exactly 1/13 as it was before all this hocus-pocus.

But problems in finite probability are not always trivial. Some seem to yield only to indirect methods of attack. Consider for example this question: Three person A, B, C engage in the following enterprise. First A and B play in a game of pure chance, while C sits out. At the end of the game the winner plays with C, while the loser sits out, and so forth, the winner in each game plays next with the third person who during the game just finished has been sitting out. The play terminates when a player wins n consecutive games. What is the chance that the play will continue through at least m games? Or consider the following.

An urn contains n balls of which exactly m are white. On drawing a ball each of the n are equally likely to be drawn. There are additional white balls of the same sort in an open basket. I draw a ball from the urn, and place it in the basket, and then put a white ball from the basket back into the urn. What is the probability that after r such operations there shall be exactly s white balls in the urn. The solution given by Burnside is

$$\frac{(n-m!)}{(n-s)!\,(s-m)!}\left[\left(\frac{s}{n}\right)^r-(s-m)\left(\frac{s-1}{n}\right)^r + \frac{(s-m)\,(s-m-1)}{1\cdot 2}\left(\frac{s-2}{n}\right)^r - \cdots + (-1)^{s-m}\left(\frac{m}{n}\right)^r\right].$$

Compared with such a problem, the following is easy. At a certain celebration there are n boys in uniforms each with a cap. The caps are thrown wildly into the air, and then each grabs the first one he finds. Assuming complete random chance, what is the likelihood that at least one boy gets his own hat. The answers is

$$1-\frac{1}{2!}+\frac{1}{3!}-\cdots+(-1)^{n-1}\frac{1}{n!}$$

Hence if n be indefinitely large, the probability that no boy gets his own hat is sensibly equal $1/e$, or about 0.368.

Let us drop these finite cases except insofar as they cast light upon what is intended by the probability of the outcome among n possibilities, as n increases to infinity.

What is meant when I say that if I take a number from 1 to 10, the probability that it is even is one-half? Of course I might like 7 so well, that I always choose it, and so the chance of my choosing an even number becomes in effect, zero. But that is not the point at issue. One merely means that half of the positive integers in the range from 1 to 10 inclusive are even, half are odd. Thus 2, 4, 6, 8, 10 are even, 1, 3, 5, 7, 9, are odd. In particular one can say much more definitely in this case that the numbers in the order of sequence are alternately odd and even. It happens to be true that the probability of choosing a prime in this range is also one-half. Indeed 1, 2, 3, 5, 7 are prime, 4, 6, 8, 9, 10 are composite. Here the numbers are not alternately prime and composite.

Similarly if in a finite sequence exactly every third number has a certain property not possessed by the others in the sequence we say that one-third of the numbers have this property and the probability of selecting a number with this property is one-third. Consider for example the property of being exactly divisible by three, for integers in the range from 1 to 12 inclusive. Here 3, 6, 9, 12 are divisible by 3, while the other numbers 1, 2, 4, 5, 7, 8, 10, 11 leave remainders on dividing by 3.

Let us turn now to the infinite sequence of all positive integers. What is the prob-

ability that a number picked at random from this set, is even. One replies of course that this probability is one-half. Indeed the numbers alternate odd, even, odd, even, like the "She loves me, she loves me not," of the romantic daisy. Again if all the numbers in the range of integers from 1 to $2n$ inclusive, where n may be as large as desired, be sorted by any means into odds and evens, exactly n are odd and n even. All this sounds very simple. But is the number of even integers equal to the number of odd integers in the complete list? We face at once the question of transfinite numbers. Again we might say that the probability of drawing an integral multiple of three from the set of all positive integers, is one-third, since every third number exactly is such a multiple. But consider the following sequence 1, 3, 2, 5, 7, 4, 9, 11, 6, 13, 15, 8, . . . where the numbers fall into consecutive sets of three, each set being of the form, $4n-3$, $4n-1$, $2n$. However little we may approve of this undignified scrambling of the orthodox sequence, we must admit that we have here an arrangement including each of the positive integers exactly once. Mathematically it is just as honest as the usual uniformly increasing order. But in the new arrangement exactly every third number is even. Hence the probability of picking an even number seems now to be one-third. This simple question would suggest first a study of transfinite numbers, and secondly the practice in the infinite sets of saying always "limit as n increases toward infinity."

Let us next ask what is the probability of a positive integer being a prime, or more precisely what is the limit of the ratio of number of primes less than n, to n itself, as n approaches infinity? Now a casual inspection of a table of primes suggests that the primes are distributed, is an almost unpredictable fashion, but tend to thin out as we progress through higher numbers. Even an approximation, for n finite is hardly trivial, but it is known that for n large, the number of primes less than n is aproximately expressed by $n/\log n$ where $\log n$ is the natural logarithm of n. If now we pass to the limit, we conclude that the probability of a positive integer being a prime is zero. But in the finite case a probability of zero always meant mere impossibility, and certainly prime numbers are possible. Again the probability, in this sense, of a natural number being a square is zero, while the ratio of the number of primes less than n, to the number of squares less than n approaches infinity with n. This is all very disturbing to one accustomed to the finite theory only. One may go further. Of the three probabilities, 1st, that the number be less than a billion, 2nd that the number be a square, 3rd that the number be a prime, each is zero, while of the second and third each is infinitely many times as great as the preceding.

Consider the following traditional paradox. Suppose I pick two natural numbers at random, say a and b. The probability that a exceeds b is surely unity. Indeed there is but a finite aggregate of distinct natural numbers which fail to exceed b, while infinitely many do exceed b. The probability of a which was picked at random, being in the relatively trivial finite aggregate is zero. In other words it is "practically certain" in some sense, that a exceeds b. But the situation is entirely symmetrical. By a like argument it is practically certain that b exceeds a. Indeed we are faced with the dilemma of being practically certain that each exceeds the other, and at the same time knowing positively that such a mutual relationship is impossible. Indeed probability for the whole set of natural numbers appears to be a rather strange thing.

One might reasonably suppose that a sequence of actual zeros ought to have as its limit only zero. Now the probability that a given natural number, x, "taken at random" is less than n is always zero for n fixed. But as n approaches infinity this becomes the interpretation of the notion that x is a number in the sequence

of positive integers. That x is in this sequence is by hypothesis certain, so that the limiting probability is unity. One can of course explain this paradox but it confuses the beginner.

Problems on duration of play such as one I mentioned a little while ago and sequences often show practical limitations that might be overlooked at first glance. For example if a coin is evenly balanced and fairly spun at the rate of 12 twirls per minute, without interruptions, there is approximately an even chance of encountering a sequence of 10 or more heads in one hour and 58 minutes. But suppose we ask modestly for a sequence of not less than 20 heads. At the same rate of spinning continued day and night, one must count on 85 days for an even chance. But suppose one is still more ambitious and ask for a run of at least 40 heads. It would take at the same rate of spin 241,724 years to get an even chance at such a run. Thus from a practical viewpoint, a game which can be expected to bring in a fortune only for a long run of successive heads might as well be left to persons with more than the 2 seconds of patience that New Yorkers usually show.

The so-called St. Petersburg paradox is doubtless familiar to most of you. It is of interest that there is even as yet no generally accepted verdict as to how one is to treat this proposition. At the risk of being trite I shall restate its simple formulation. A coin is to be spun repeatedly. Each time a head turns up, the round terminates, the banker pays the player, and the next round is started. If there is a sequence of n tails before the head appears which terminate a round, then the banker is to pay 2^n dollars for the outcome. Thus if on the first spin, a head appears, the banker pays one dollar. If there is first a tail and the head appears on the second spin, the banker pays two dollars. If there are tails twice in succession and then a head, the banker pays four dollars, and so forth. What is the equitable price per round that the player should pay the bank for the privilege of engaging with him in this game?

Now by the first principles of probability theory one sees that the probability of heads appearing for the first time on the nth spin, is $(\frac{1}{2})^n$, and for any such round the player receives 2^{n-1} dollars. The value of a single round to the player may be reckoned as follows. He has the chance of $\frac{1}{2}$ of winning one dollar on the first spin, plus the chance of $\frac{1}{4}$ of winning two dollars on the second spin, plus the chance of $\frac{1}{8}$ of winning four dollars on the third spin, plus, and so forth. The infinite sum has each of its terms exactly $\frac{1}{2}$ dollar in value, so that the total expectation per round is infinitely valuable. No matter how large a sum the player is willing to pay for even one round, it is unfair to the banker.

From the point of view of common sense, this seems palpably absurd, and yet if there be any mistake in the theory, it has eluded mathematical examination. Numerous explanations have been offered, some of them ridiculous to most people. One probably plausible method of disposing of the paradox is to examine the effect of supposing the banker's funds large but limited. For example if the player should agree to accept the restriction of the banker's liability to one million dollars, then the value per round in this game is found to be mathematically worth only $10.95. If the liability should be one billion dollars, a fair sum even in these days of soldier-bonus, silver standard, etc., the mathematically computed worth of a round would still be less than $16. There yet remain however many peculiar possibilities in this sort of a game. But since as originally stated the banker is sure to lose no matter how much is charged per round, and the would-be players are sure to protest that even small sums are exorbitant for such a "gyp" game—the mathematical public can lean back with the comforting feeling of morals sustained —and with neither side desirous of engaging in such an unappealing enterprise.

The majority of important applications

of probability theory arise undoubtedly through use of a frequency distribution, which in convenient cases is or may be regarded as being that of the Euler-Laplace normal probability function. This, for suitable choice of units is representable by an equation of the form

$$y=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}.$$

This is a smooth symmetrical bell-shaped curve with a central maximum at the origin, and hugging the x-axis with extraordinary closeness, as x recedes toward infinity in either direction. This curve may be regarded as a limit approached by the binomial distribution if a broken line be used to join the ends of ordinates erected at equal intervals and of height proportional to the successive terms in the expansion of $(1+1)^n$. But this limit business is not entirely trivial. The wrong sort of limit curve is easily obtained.

One has a sequence of broken line graphs with successive ordinates having respectively the following magnitudes.

$n=0$, $(\cdots 0,1,0 \cdots)$,
$n=1$, $(\cdots 0,1,1,0 \cdots)$
$n=2$,$(\cdots 0,1,2,1,0 \cdots)$
$n=3$, $(\cdots 0,1,3,3,1,0 \cdots), \cdots$.

These fall into the familiar Pascal triangle of binomial coefficients. To reduce the corresponding diagrams to comparable size and suitable location, one method is to use n even and to seize upon the middle term and then to introduce a factor of proportionality to keep this of fixed magnitude. Next the interval between ordinates must be adjusted by use of $\sqrt{n}$, so as to keep the total area under the broken line, constant. To do these things one needs to know Stirling's celebrated asymptotic formula for factorial n, and to use it intelligently.

Let me draw to a close with a personal, almost a religious question: What should be our modified outlook concerning the immediate future as a consequence of an unexpected sequence of fortunate events?

1. Should we face the future with a sobered but grateful acknowledgment of undeserved good fortune and with Bunyan say as we see our less blest neighbors, "There but for the grace of God, goes John Bunyan."? But granted that we should, the question still remains open as to what we ought to expect will happen next.

2. We may adopt the view that there is eventual compensation. We cannot hope to draw aces always from the pack, and if we have drawn more than our share to date, then the future will balance the account. This view is a great comfort to those who feel themselves unjustly oppressed. "The bigger they are the harder they fall." "Pride goeth before destruction." "The meek shall inherit the earth." "When Dives shall moan for a drop of water, Lazarus will be reclining in Abraham's bosom." "You can't eat your cake and have it too." "Every cloud has its silver lining." "The good die young." "Lightning never strikes the same place twice." "Opportunity knocks but once." etc. Although certain problems in finite probability give encouragement to views of this sort, there seems no reason to regard such ideas as ultimate.

3. Should we accept a favored past as indicating a changed probability for our immediate future? Where one has found gold nuggets, is it worth looking for more? Is a tried recipe better than a random mixture of raw food products? Is it not true that to those who have more shall be given, and that the best promise for future success is to succeed? Whose artistic work is likely to be more valuable, that of the trained and experienced artist with numerous masterpieces to his credit, or that of the amateur about to complete his first production? In short, if our dice keep showing six-spots, are we not properly entitled to judge them to be weighted and plan accordingly?

4. Should we count on cycles? Are we to ride the crest of a wave of good fortune while we may and then anticipate a limited period of depression? Does the alter-

nation of summer and winter with uncertain hot spells, Indian summers, and early frosts indicate the way Nature treats us in general? Styles come and go, often to recur again, but is all existence essentially periodic, like the sun spots or the repeated ice ages?

5. Are we to expect despite unfortunate lapses, essential improvement all along the line? Are people on the whole happier, wiser, braver, more generous, than in times past? Is the gradual prolongation of mean human life, the gradual extermination of wild beasts, the steady spread of literacy, and increase in travel, is this characteristic of existence as a whole? Is a richer prosperity the reasonable hope, not for ourselves alone but for everyone?

In regard to such matters, the mathematician like others must make his working decisions. He cannot turn to a table of formulas, or a computing machine. He may feel the kindly hand of a paternal Providence, or with stoic fortitude face the harsh bludgeonings of Fate. Perhaps he is reduced to saying with Omar Khayyam:

"There was a Door to which I found no key,
There was a Veil past which I could not see;
Some little talk awhile of me and thee
There seem'd—and then no more of thee and me.

But this I know; whether the one True Light
Kindle to Love, or Wrath consume me quite;
One glimpse of it within the Tavern caught,
Better than in the Temple lost outright.

The Moving Finger writes: and having writ
Moves on: nor all thy piety nor wit
Shall lure it back to cancel half a line,
Nor all thy tears wash out a word of it.

But leave the Wise to wrangle, and with me
The quarrel of the Universe let be;
And, in some corner of the hubbub croucht
Make game of that which makes as much of thee."

Some Observations on Teaching Percentage

By Charles H. Butler
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Percentage is without doubt one of the most widely used applications of arithmetic. If, on the grounds of universal practical direct utility, a case can be made for any part of secondary school mathematics, then certainly such a case can be made for percentage. Without some understanding of it, it is impossible to perform such simple and universally useful activities as computing interest, determining discounts, and making simple routine comparisons of quantitative data. Even the intelligent perusal of the daily paper requires an understanding of percentage. Aside from the fundamental processes themselves, percentage probably has more widespread and direct applications to the ordinary affairs of all people than any other special topic in the whole range of mathematics.

If we may postulate this, and if we may also postulate the validity of the practical utility objective, so generally and so vigorously advocated in these days of curriculum re-examination and revaluation, then certain conclusions are inescapable. If we are to be consistent about the matter, it must follow that every reasonable effort should be made to bring the teaching of percentage to a high degree of effectiveness:—to such a degree of effectiveness, indeed, that a majority of students who come into the upper grades or the secondary school will have some genuine functional understanding of percentage.

That our present instruction does not, on the whole, attain this desideratum will be apparent to any who care to investigate the matter. Some time ago, to check this, the writer constructed a test consisting of ten problems involving various applications of percentage and administered it to a high school class in commercial arithmetic. The problems were stated in words. They involved no computations of unusual difficulty, the aim being to test the ability of the students to analyze and set up the problems. The average score on the test was 3.4 problems correctly solved.

Equally striking is the case of two students, both sophomores in college, who, upon being given a series of numerical ratios and asked to translate these into per cents, confessed to the writer that they had no idea of how to go about it.

Here, then, we have an anomalous situation. A subject that is admittedly of great practical importance is taught with so little effective result that many, perhaps most, individuals to whom it has been taught find themselves in the position of being unable to apply it effectively. It is probably true that most people know and can use the fact that "50% of any quantity equals half of it" and a few other similar fraction-per cent equivalents. It is probably true, also, that most people who have studied arithmetic through the sixth grade or beyond can solve set problems such as "Find 43% of \$120." However, the ability to apply a few common equivalents or even the ability to determine a given percentage of a given base by no means implies any considerable degree of analytical insight into the relations involved in percentage generally. On the contrary, the writer is disposed to feel that such applications as have been mentioned may often be based on dogmatic rules and verbally memorized equivalents rather than upon any rational analysis of relations involved.

The situation presents a challenge to all who are engaged in the teaching or supervision of arithmetic in the upper grades and the junior high school, or who are concerned in any way with the organization of the courses in arithmetic at the upper grade or secondary school level. It will be of interest to carry the examination

further in order to see if any plausible reasons for the obvious ineffectiveness can be advanced, and if so, what measures might be instituted to make the instruction more effective.

Perhaps many theories could be advanced to account for the evident ineffectiveness of the teaching of percentage. In the opinion of the writer, however, one or more of three explanations must cover the field of possible explanations, provided we may postulate a willingness to learn on the part of the normal student. If students are normally intelligent and if the subject matter which they are expected to master is not of a difficulty beyond their ability to comprehend, and if it is properly and intelligently presented to them in their textbooks and by their teachers, then it appears obvious that at least a reasonable degree of mastery of such subject matter will be attained by the students.

The implication of the foregoing statement is simply this: either (1) the majority of students are not normally intelligent; or (2) mastery of the subject of percentage involves functional relationships which require intellectual maturity beyond that of the normal secondary school student; or (3) the subject of percentage is not presented to the students as rationally and intelligently as it might be in their textbooks or by their teachers. Now let us examine these three possibilities further and see if a process of elimination will lead us any nearer to a conclusion.

First, can we suppose it possible that any considerable majority of students in the upper grades and secondary schools are not of normal intelligence? Admittedly this might be true in certain schools or in certain sections of a community or even perhaps in certain whole communities. But it can hardly be regarded as possible when we are considering the population of the schools as a whole. *Our very definition of normality denies it.* A standard below which a majority of a group is to be found is not a norm for that group. We may, therefore, eliminate the sub-intelligence argument from the list of plausible explanations of the almost universal ineffectiveness of the teaching of percentage.

Second, can it be that percentage is just naturally too difficult a subject for students in the upper grades and the junior high school? That is to say, does it involve relationships that are too abstruse for the students to grasp if they are of normal intelligence and if the concepts are properly presented to them? Upon reflection one can hardly think so. The essential relationships that are involved are really relationships among ordinary numbers and are precisely those that are involved in any simple problem of buying and selling. There can be no doubt, however, that children are much more proficient in analyzing and solving such problems than they are in analyzing and solving corresponding problems in percentage.

If we may regard the above argument as having disposed of the "inherent functional difficulty" hypothesis and if we may assume that the three hypotheses which were advanced completely cover the possibilities of explanation of the ineffectiveness in the teaching of percentage, then are forced to the conclusion that at least a major part of the explanation lies in the third hypothesis, which was that the subject is not properly and intelligently presented to the students in their textbooks and by their teachers. The examination and development of this third hypothesis with its implications will occupy the remaining portion of this paper.

It is the firm opinion of the writer that the subject of percentage is presented in nearly all textbooks in a manner that is far more involved than is necessary. One fails to find in the textbooks any consistent emphasis upon a simple functional relationship, freed as far as possible from confusing elements. Instead, it almost seems to be purpose of textbook writers to include, on the score of "completeness" as many of these confusing elements as possible. Invariably, one finds percentage inextricably mixed up with decimal equiv-

alents, fractional equivalents, rules of operation, "cases," short cuts, and what not.

The objection which is raised here is not that these relations and rules and short-cuts are non-existent nor that they are illogical, but simply that they are for the most part unnecessary and are, in fact, serious impediments to clear relational thinking about per cents, *especially in the early stages of instruction.* For example, why should it be expected of a young child that he learn to recite and to write that there are half a dozen or so ways of expressing 93 per cent of a number? Yet, the textbooks advise him that 93 per cent of a number may be written as 93 per cent$=93\%=.93=93/100=93$ hundredths. Certainly it may, but what then? That a child has learned to write or recite this series of equivalents can hardly be taken as any guarantee that he has learned to think functionally about per cents and the different elements involved in percentage problems.

Then take the matter of the various "cases" of percentage. The child is taught that such and such a type of problem is a problem involving case 1, and that it is to be worked in such and such a manner. An illustration is given, a rule stated, and a set of examples involving "case 1" given for practice. The child refers to the illustration, follows the rule, and works the examples, probably without much difficulty other than the labor of computation, and probably without much real understanding of why he has done what he has done in the way he has done it. A little later a similar procedure is followed with reference to "case 2," and still later with "case 3."

Then, with the introduction of a mixed set of problems including all three types, the real trouble begins. And it is not in computation, imperfect as that is often found to be. The real difficulty is that the students are often unable properly to classify the problems for themselves—to determine whether this problem is a "case 1" problem or that one a "case 3" problem. In short, the trouble is that in studying cases as such, the students are not effectively given a single unifying concept by means of which they can analyze the relations, given or implied, among the elements in the problem, and as a result of such lack of analytic insight into relations of the elements, the most gross errors in method continually occur—errors that indicate beyond any doubt a pretty complete lack of any real understanding of the problems.

Now what, if anything, can be done about it? Can some unifying concept be found which will permeate the whole range and gamut of percentage and which is capable of being understood by normal pupils to the extent that it will help to clarify their analyses of all situations involving percentage? The author believes that such a concept can be found, and that it is the concept of *a single per cent of a given base as an entity*—the concept of *one per cent of a given base as a whole unit itself.* And he here proposes the hypothesis that if pupils can be taught to think consistently of one per cent of a quantity *as an entity*, then a very considerable part of the difficulty which they encounter in attempting to analyze problems involving percentage will disappear.

Experimental evidence of the correctness or incorrectness of this hypothesis is lacking here, because so far as the writer is aware, no scientific investigation of this particular problem has been published. However, his experience of several years has convinced him that it is quite possible to teach normal seventh grade children this concept, and that it does help them in the analysis of the relations that exist among the elements in various types of percentage problems.

The reasoning leading to the hypothesis seems clear and sound. (1) It is perfectly reasonable to assume that children on the whole are more familiar with whole numbers than with fractions or per cents. (2) It is also reasonable to assume that a child will be able to analyze a situation involving familiar elements more easily

than a situation involving elements which are not familiar to him. (3) By thinking of *each per cent as a unit itself* the concept, for example, of 93% of a given base is transferred from the field of fractions to the more familiar field of whole numbers. That is to say, 93% (a part of something) becomes 93 per cent*s* (plural) or 93 parts of the same thing, each of the 93 parts being thought of as a whole little something itself.

It would, of course, be idle and false to maintain that the translation of the foregoing hypothesis into practice is an easy matter or that it can be done in a hurry. It involves some very real difficulties that should not be minimized. Three of these will be mentioned here.

The first is the unfortunately unavoidable language or terminology of percentage. Usually the first thing a child learns about percentage is that "per cents mean hundredths" or that "a per cent is a hundredth." Thus the earliest concept of a per cent which the child gets is that it is a fraction. This association, of course, tends to persist and to grow stronger unless immediately elaborated by the explanation that even a part of anything may be regarded as a *whole* part, or a sort of a sub-whole itself. When children come to the seventh grade with the firmly fixed idea that a per cent can be thought of only as *a fraction*, it requires much time and patience to make the above mentioned elaboration clear to them. The expression "93 apples" connotes immediately and without any difficulty a definite number of whole things, but largely because of fixed language bonds. The usual connotation of "93%" is merely that of a part of one whole thing. To very few pupils indeed does it connote also *93 whole parts* of one big whole. Yet, by persistent and well directed effort, this connotation can in most cases be attained.

A second difficulty has already been implied but is worthy of special mention. It is that certain fixed associations must be broken down or shunted into the background in order that the new concept of a per cent may become paramount. These associations or habits of thought usually have been brought about by somewhat intensive drill, and the matter of readjusting them and fitting them into the new scheme of thinking is not an easy one. Personal experience and observation, however, have convinced the writer that it is quite possible.

A third consideration is that more time is required to teach pupils to analyze their problems in terms of single per cents than is required to teach them how to "do" examples by the methods of "cases" and rules. Results will not be so quickly attained, but it is believed that they will be more permanent and, most important of all, they give more promise of attaining the fundamental objective involved—the ability to make discriminating analyses of the relations involved among the elements in the problem. Speed should be distinctly a secondary consideration anyway, and the fact that this proposed plan (of making the single per cent the basic unifying concept in all percentage problems) may operate somewhat less rapidly than other plans in use should not weigh too heavily against it.

In conclusion, it appears that the proposed hypothesis is based on clear and logical considerations and it is believed that in the hands of a competent teacher it is quite feasible. Experimental work is needed, however, to test the hypothesis. It is to be hoped that such experimentation will eventually be undertaken on this very practical problem of instructional organization and technique.

NOTE: The reader should consult Edwards' excellent report entitled "A Study of Errors in Percentage," published in *29th Yearbook of the National Society for the Study of Education, pp. 621–640* (1930). Substantiation will be found therein for certain points made in the present article. Edwards also makes certain statements that at first appear to be in conflict with statements made in the present article, but careful study shows such conflicts to be more apparent than real.

A Brief Professional Philosophy for Teaching of High School Mathematics*

By H. F. Munch
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One of the most important factors in success in teaching is the attitude of the teacher toward his job, toward the pupils, and toward the subject which he teaches, in short, his viewpoint with regard to these factors in the learning situation. You no doubt remember the Biblical quotation (Proverbs 29:18) "Where there is no vision the people perish." Surely where the teacher has no vision as to his responsibilities, his opportunities, the possibilities of his job, if he has no vision as to the beauties of mathematics, its power, its eternal verity, its universality, its great value in the process of educating young people as citizens of our republic, as members of a family group, for their vocations, or to develop ethical character, the pupils perish. The teacher who has no vision as to the value to the state and to the local community of the young people who come to him for instruction, the teacher who does not sense the wonderful possibilities enshrined in them, who does not comprehend the anxiety of parents and friends for their success and welfare has not the vision to be a teacher.

The teacher must remember that the pupil is the sacred vessel for which the school exists. It is the great responsibility of every teacher to make some contribution each day to that motive force which will raise each pupil with whom he comes in contact to a higher plane than the one on which he stood before. The teacher should therefore, through his subject, through his personality, and through his wisdom and skill in teaching his subject strive to incorporate in the pupil those knowledges, attitudes, habits, ideals, appreciations, and powers that will fit him to be successful in life and to enjoy life to the fullest. Again, the pupil is the all important factor in the situation. His real welfare should be the mathematics teacher's deepest concern. Whatever makes a worth while contribution to his mental, physical, or spiritual growth or development should be of the deepest interest to the mathematics teacher. He should be especially concerned about those knowledges, attitudes, habits, ideals, appreciations, and powers which mathematics may best aid the pupil in acquiring and he should be ever alert to develop ways of getting the pupil to acquire them. This, of course, includes the practical value of mathematics to the pupil in the solution of problems of a mathematical nature; but it also includes the building up of those habits, ideals, and traits of character which mean so much in training for citizenship, for worthy home membership, for a vocation, or for the happiness of the individual in life.

The mathematics teacher who is to do this must know his subject. It is axiomatic that a teacher cannot teach what he does not know. Furthermore, he can have little enthusiasm for that which he does not know and understand so well that he sees its applications and its values to himself and his pupils. If he can have no enthusiasm for his subject, how can he inspire his pupils with that enthusiasm which will impel them to master the subject which many of them find difficult?

Besides all this the mathematics teacher should have been and should continue to be a student of the psychology of learning. He should know *how* pupils learn mathematics best, and that a real genuine under-

* Read at the annual meeting of the Mathematics Section of the State Teachers Association at Raleigh, N. C., March 20, 1936.

standing by the pupil of the principles and processes involved are essential. He should therefore develop techniques of presenting mathematics material to boys and girls in such a manner that they can easily understand it. He should also know that the pupil must work many problems even after he understands the principles and processes in order to come into permanent possession of them.

Every teacher should recognize the value of *interest* in the learning process. It is suggested, therefore, that every mathematics teacher use every means at his disposal to make mathematics interesting and to give life and meaning to his work. If you ask how this may be done, the following suggestions are offered as means to this end:

1. Use the history of mathematics. Besides creating interest in mathematics this should also broaden and deepen the pupil's knowledge of the subject.

2. Have reports on the contribution of mathematics to the progress of mankind.
 a. To the development of science.
 b. To the development of engineering.
 c. To the development of architecture.
 d. To the development of aeronautics.
 e. To the development of improved methods of national defense.
 f. To the development of improved methods of business and finance.

3. So present the work that the pupil will see the application of mathematics to the every day problems of life. It is suggested that the teacher invite the pupils to bring to class all sorts of problems arising in the home, the shop, the factory, the business or the civic life of the community and that the teacher supplement wherever advisable the material of the text used with that found in other texts.

4. The mathematics teacher should endeavor to get the pupil to realize the contribution which mathematics makes toward understanding quantitative facts as expressed by speakers and writers. This necessitates an understanding by the pupil of the vocabulary of mathematics and such commonly used mathematical tools as the graph, the formula, mathematical symbolism, and the equation. Every mathematics teacher should use the technical vocabulary of the subject correctly not just when its use is demanded but as much as the occasion will permit. This is important because we think in terms of words as well as concepts. Hence a knowledge of the vocabulary of mathematics is necessary. The pupils' interest in the value to them outside of mathematical work might be stimulated by having them make lists of words found in ordinary reading that belong to the vocabulary of the subject.

5. Pupils should be encouraged to bring to class clippings from newspapers and magazines which pertain to mathematics. These clippings should be given prominence by comments in class or by placing them on the bulletin board in the mathematics class room.

6. In more advanced courses those pupils who are mechanically inclined may be encouraged to make certain mathematical instruments either for use in the class room or for use in doing field work. If carefully guided this field work may become the basis for problems that will be more real to the pupil than any textbook problem.

7. Pupils should be encouraged to dramatize bits of mathematics or to put on mathematics plays.

8. Some pupils may be easily interested in mathematical puzzles, fallacies, and catch problems. These should be used as recreations, not as the problem material for class work.

9. It is a splendid exercise to have pupils write themes on the values to be obtained from a study of high school mathematics.

10. The mathematics teacher should ever remember that it is human nature to like to do best that in which one can excel. This is true of mathematics. The teacher should therefore insist that the pupil get a thorough understanding of mathematics as he goes along and thereby engender a love for the subject "per se," which is one of the best ways to insure interest in and study of the subject.

The high school teacher of mathematics should realize that the mathematics most often used in life by the rank and file of people is arithmetic. No matter how much other mathematics the pupil may know if he has not a thorough mastery of the fundamental processes and principles of arithmetic and if he is unable to apply this

knowledge to the every day problems of life the public will criticize severely, and justly so, that school or teacher that turns out such a product. To develop this thorough mastery of arithmetic in his pupils should be, therefore, one of the primary aims of the high school teacher of mathematics throughout the entire high school course. To this end every mathematics teacher should lose no opportunity to secure this complete mastery of arithmetic through the medium of algebra, geometry, trigonometry or any other mathematics taught. No matter who is at fault for the deficient knowledge of arithmetic of a high school graduate, the high school teacher will be held responsible by the general public. Hence the high school teacher must in self-defense be sure that his pupils know arithmetic.

There is a tendency at present on the part of some people to abandon many of our splendid heritages in the way of institutions and customs such as the home, religion, neighborliness, conversation, and mathematics. Because of this tendency the teacher of mathematics should be prepared to champion the cause of mathematics as educative material. He should be able to give to either parents or pupils the values to be derived by the pupil through the study of mathematics. He must remember that a pupil who sees little or no value to be obtained from the study of mathematics is apt not to spend much time in its study and hence is a potential problem in the mathematics class room. In short, the mathematics teacher should be prepared to sell his subject to the pupil, the community, and to the school authorities.

The mathematics teacher should wherever possible give his class room a mathematical setting by equipping it with all apparatus needed for the efficient teaching of the subject. Such equipment as blackboard compasses, straight edges, and blackboard protractors should be in every mathematics class room. There should also be pictures of some of the great mathematicians, of applications of mathematics in art, architecture, or in industry. The room should contain work done by the pupils such as graphs or good papers on the subject or mathematics material collected by them. Mathematics posters also serve a good purpose.

In all school work time is a relentless factor. Consequently, there should be no wasted time in the mathematics class room. Methods of teaching should be carefully selected so as to take as little time as is consistent with real understanding on the part of the pupil. Whatever projects or devices are used should only be chosen if they will serve to motivate the work or if they will secure the desired learning products in the least possible time. It can not be expected that high school pupils should see the importance of time since they are apt to think that time is the one thing of which they have an unlimited amount. This is all the more reason why the teacher should be cognizant of the importance of time in the educative process. He should also attempt in this connection to have his pupils develop efficient habits of work and reasonable speed in computation.

The mathematics teacher should leave nothing to chance in securing the desired learning products. When he teaches any bit of mathematics, he must have all the objectives of the teaching of that topic clearly in mind and then so teach as to attain those objectives in the least possible time. He must not take any thing for granted. He must not assume that pupils know all they have been over in the courses taken thus far. Only when adequate inventory tests are given will he really know the status of the pupil's mathematical knowledge when that pupil comes to him. Furthermore, the teacher must not assume that because he has tried to teach certain work, that the pupils have mastered that work. Only after giving adequate tests may he be sure that the pupil has really mastered what has been taught.

In this connection it would no doubt be appropriate to emphasize the importance of reteaching to many pupils in learning mathematics. Reteaching is essential whenever principles or processes are not completely understood. Christ said, "For ye have the poor always with you." It is a safe assertion that the mathematics teacher will always have with him some pupils who are "poor" in his subject. To them reteaching is a necessary step toward mastery.

The mathematics teacher should also understand the psychology of drill and the place of drill in the learning process. He should realize that the permanence of the pupil's knowledge of many processes and principles in mathematics depends upon the amount and effectiveness of drill to which the pupil is subjected. Few pupils acquire the necessary skill without much practice in those operations which are so basic to mathematics work that pupil's responses to them should be automatic. Because of this, a part of almost every class period should be spent in drill even though the time allotted each day to this phase of work may be small.

No professional philosophy for teachers of mathematics would be at all satisfactory without emphasis upon the professional attitude on the part of the teacher. If he has this professional attitude, through reading and study, through attendance at meetings of mathematics teachers, through summer school attendance, and in other ways, he will gradually build up his own professional philosophy. If he does not have this attitude there is little hope that he will ever build up for himself a satisfactory one.

Every teacher of mathematics should own at least a half dozen good books on the teaching of mathematics. Every mathematics teacher should belong to the National Council of the Teachers of Mathematics, take the MATHEMATICS TEACHER and should belong to the mathematics section of the state teachers' association of the state in which he teaches. Every mathematics teacher should be professionally minded to the extent of reading the history of mathematics. Furthermore, he should make a thorough knowledge of the history of the subject he teaches a vital part of his professional equipment.

The mathematics teacher should assume at least a part of the responsibility for building up the mathematics section of the school library. He should see to it that it includes good books for both himself and his pupils. There should be books on the teaching of mathematics, history of mathematics, mathematical recreations, and other good books of a mathematical nature.

One of the great aims of the teacher of mathematics should be to develop skill in quantitative thinking. By this is not meant skill in mathematical manipulation but rather in being able to really sense the significance of the quantitative relationship involved in a problem situation and in the use of these relationships to solve the problem. Inability to do this is a deficiency almost universally found among even the graduates of our American high schools. One of the major aims of our teachers of mathematics should be to remedy this serious shortcoming.

We make no apologies for emphasizing certain disciplinary values for which the teacher of mathematics should strive. Many mathematics teachers make no effort to secure these values. Indeed if we should judge by their teaching we would conclude that they were not even aware of them. By its very nature, mathematics is the best of the high school subjects for developing certain attitudes such as accepting responsibility for the correctness of a result, an attitude of work, an attitude of inquiry, an attitude of obedience to law, and so on; certain traits, such as accuracy, neatness, persistence, orderly arrangement of material; certain ideals, such as truthfulness and obedience to law; certain abilities, such as the technique of problem solving or reasoning; and certain appreciations, such as the appreciation of

the heritage of this generation from our ancestors, the appreciation of the power of mathematics to solve quantitative problems, of the beauty of a mathematical solution and of geometric form. This matter deserves a great deal of attention from the mathematics teacher because one's attitudes, traits, appreciations and ideals either determine or are the measure of what a person really is. It might be put this way. It matters little how much mathematics you teach a youth if in so doing you permit him to develop wrong attitudes, ideals, traits and appreciations. The value of these to youth and to society transcends the value either to him or to society of a knowledge of mathematics.

David Eugene Smith emphasizes the importance of these indirect values of the study of mathematics as follows:

> It seems timely to say at this point that no school curriculum in whole or in part can afford to base its planning too completely on present demands for specific uses. It is not possible to foresee conditions in the world for any considerable time, and therefore, general preparedness, openmindedness, and resourcefulness may prove to most of the population more important than overspecialization.[1]

There is another bit of philosophy which is very important for the mathematics teacher. It should be emphasized at present because there is so marked a tendency for most people and particularly young people to have an aversion to anything that savors of work. There never has been developed any means by which a pupil may get the values that should be obtained from the study of mathematics except work on the part of the pupil. Pupils learn to solve problems by solving problems. They develop accuracy and speed through practice. They learn to reason by reasoning about things that come within their knowledge and experience. They develop mastery of processes and principles by much application of these processes and principles. It is recommended, therefore, that whenever advisable a part of the class period be devoted to supervised study. Even though this be done it seems evident that the pupil must be expected to study some outside of class, unless the class period is materially lengthened, if he is to thoroughly fix in mind those principles and processes explained in class. Without this study by the pupil, assimilation, which is so important in the learning process, is impossible. The amount of time the pupil studies his mathematics outside of class should of course be dictated by circumstances, but should be in the neighborhood of one hour per day.

Dr. A. Lawrence Lowell, President of Harvard University, speaking before the Department of Superintendence of the National Education Association at Boston in February, 1928, emphasized the importance of effort on the part of a pupil by the use of a formula. The formula he gave was $(x+1)yc=p$. He assigned values to the letters of this formula as follows: $p=$the benefit obtained by the pupil, $x=$ the quality of instruction, $y=$the mental effort of the pupil, and $c=$the mental ability of the pupil. Dr. Lowell explained this formula as follows:[2]

> I write the formula thus because if there be no effort, voluntary or induced, by the pupil or if he have no intelligence of any kind there can be no education and the product is zero. Whereas without teaching above the elementary stage extraordinary personal effort and great natural ability produced the education of a Benjamin Franklin. In short, there may be self education without teaching but there can be none without effort and intelligence on the part of the pupil, however good the instruction. Nor is the formula in other ways inaccurate, for a doubling of the pupil's effort doubles the effect of good instruction and the better the teaching the greater the earnestness of the pupil's effort.

Have we not in the past decade or more thought too much about enlarging the product p by increasing the x (quality of

[1] Smith, David Eugene, *Second Yearbook* of National Council of the Teachers of Mathematics. Page 253.

[2] Lowell, Dr. A. Lawrence, Department of Superintendence, *Official Report*. Boston, Mass., Feb. 26–March 1, 1928. Page 13.

instruction) with comparatively small attention to the factor y (pupil's effort)? Since c is the intelligence of the pupil and cannot be very greatly increased, there are two controllable factors in making p large. Those are x (quality of instruction) and y (personal effort and concentration on the part of the pupil). Every mathematics teacher should therefore see to it that his teaching is as good as it can be made but he should also devise ways and means of securing maximum personal effort on the part of the pupil, for without this effort there can be no education. Let us therefore recognize the importance of *work* in the learning process.

The teacher should remember that mathematics is not a tool but a mode of thinking. Dr. Judd, in the *Third Year Book* of the National Council of Teachers of Mathematics has made this very clear. He says it is not merely a tool which we pick up when we wish to solve a problem and which we lay down again when the problem is solved. Rather it is a mode of thought, which when once acquired can never be laid down. This mode of thought is one of the main differences between primitive people and those highly civilized. Hence mathematics is appropriate material for high school pupils to study.

Because of the very nature of mathematics many pupils have trouble in mastering it. Nor is it sufficient for the teacher to merely say that the trouble is due to lack of intelligence or lack of application on the part of the pupil. In certain cases it may be true that the seat of the trouble lies in one or both of these causes. In other cases it will be found elsewhere. Whatever the cause it becomes a major function of the mathematics teacher to diagnose the difficulties of his pupils with the utmost care. If he knows little or nothing about such work he should acquaint himself through reading and study with the various techniques that may be followed in performing this important function. Having diagnosed the difficulty he should attempt to remove the cause and prescribe the remedy. Again, if he is not familiar with remedial procedures he should perfect himself in this difficult art.

Last but not least, every teacher should be most vitally concerned about the morals of his pupils. The mathematics teacher should be concerned even more about the morals of his pupils than concerning their ability to solve problems. This is true because there is no citizen so dangerous as one who is lacking in this regard. He has no place in the commercial or industrial world. There is no place for him in the professions. The world knows that such a person is a menace to society and that there is no place for him at all. Hence, the mathematics teacher should use his subject for character training.

It is hoped that the phases of this subject here treated may stimulate the mathematics teacher to build up a professional philosophy that will make him a better teacher of the subject, one who is both happy and efficient in his work.

Finding Social Mathematics in School Activities

By Dorothy Noyes
Tappan Junior High School, Ann Arbor, Michigan

Judging from a number of the recent articles on high school mathematics it would seem that there is much thinking and considerable experimenting being done on the subject of making mathematics more functional. Mathematics has its definite place in our school organization outside of the immediate classroom; a place proportionate to that of other school subjects but which perhaps has not been as evident.

For instance, if we were considering how active a force the study of English was in any school set-up, we could readily point to its use in the school paper, the school magazine, public speaking activities and many dramatizations. Social Studies is useful in governmental activities and Art in decoration, posters and stage settings. Home Economics and Practical Arts are also manifest and tangible in articles actually constructed as a result of class training. Music is constantly evident and, to many of the pupils, Physical Education is the very essence of school activity. Mathematics, too, makes a well defined contribution to school organization but it does seem as if this contribution could come, more than it ordinarily comes, through the mathematical activities of the pupils themselves.

There are phases of every semester's course of study in the junior high school which lend themselves well to the utilization of the mathematics of school activity. When relevant, this material should evidently be used to advantage in any grade. The eighth year course of study, including its organization of stock companies for the selling of materials, has been well discussed in many articles and I don't intend to go farther into that here. Graphing in the seventh grade should and does lend itself well to picturing of school experiences and so do game scoring and batting averages and many other percentage problems.

What I wish to speak of particularly is, however, a class which we have been organizing in the ninth grade each semester for the last few years. This is a service class formed for the dual purpose of mathematical service to the school and the further study of mathematics based upon this school service.

This class is usually composed of between twenty to thirty members the majority of whom enroll in the class for one of two following reasons:

1. They do not expect to be able to go on with their education probably not even through senior high school.
2. They have been very weak in mathematics and are taking an extra semester of fundamental mathematics before beginning Algebra 1.

Some of course are enrolled for other reasons. Perhaps it is thought advisable to place a sixteen year old unadjusted pupil who has only enough credits to put him in a 7A mathematics class in this service class where he can be tried out in a new capacity. Or perhaps a pupil who definitely expects to train for business may begin his work here.

Any of these reasons, however, seems to bring the same type of pupil into the class. This is the pupil with the low I.Q. (our present class ranges from 78 to 112—there being only three with I.Q.'s of 100 or over) who has been struggling with mathematics all through his junior high school career and whose average mathematics grade in junior high school is usually better than a "D." These persons are usually also, types which are not accustomed to succeeding, are seldom given offices of importance or any responsibility

whatsoever. Oftentimes, too, they are definitely the discipline problems of the ninth grade.

Because of the types of pupils enrolled in this course and because of their purposes in taking it, it has seemed to us that the course should consist of:

1. Mathematics which is concrete and where the results of computational errors or fallacious reasoning show themselves in real mathematical situations and can be brought home to the pupil in those situations.
2. Mathematics dealing with the useful fundamental processes (in which these pupils are notably deficient).
3. Mathematical experiences in the home, school and business world of the average man—of which each child in the class is, or must soon be, a member.

In setting up our course of study for this class, it has seemed that as the school is the business organization of which the child is a member, (at least he has few other business affiliations) there should be in school activities a great many mathematical situations in which the children rarely participate but could very advantageously do so.

Of these situations in which our ninth grade class is a real service class I will mention the following.

At the beginning of every year, each child in the school is expected to buy, from the school, two padlocks at 75¢, and a social studies fee ticket at 50¢ each. He may also buy a ticket for Practical Arts at $1.00, a student activity ticket for 50¢, a ticket for 7B English fees at 50¢, and a ticket for music lessons at $2.00. This money is all being handled by this ninth grade mathematics class. We have tried it in several ways and this fall it has worked out very well. A pair of class members has been assigned to each homeroom and takes care, exclusively, of the sale of tickets in that room. Each makes a complete statement every day of the number of tickets sold, write a deposit slip, a receipt and a receipt stub for the amount of money received. The head banker checks each deposit slip with the money with me and keeps the total day's record for the class. He also takes the money, with supervision, to the central finance office for deposit and is responsible for checking receipts coming back from there. Each ticket salesman is responsible for the sale in his assigned homeroom and must keep a constant check with the adviser of that homeroom on the sale therein. He must make a weekly report showing amount of money taken in and tickets sold each day. He must also check the amount of money which belongs in each fund—according to the kind of ticket sold.

This ticket selling—we have found—gives excellent practice in drill in addition and multiplication. Here are additions which must be gone over until they check, with a score of possible errors for each total. Within reason each pupil must find his own mistakes and this necessitates a goodly amount of re-multiplication and re-addition—especially at first. This sort of mathematical practice gives these children a feeling of responsibility and necessity for correct results which they do not usually feel so strongly in any "book" problem in mathematics. Accuracy and honesty have to go hand in hand and this should be noted by the pupil. If he does honest multiplication and addition and makes honest change he'll have no trouble in making his accounts balance.

This selling and handling of money is a great game—and there are few who do not in some measure enjoy it. It is interesting to see the responsibility that most of the people who have made very poor previous records in mathematics (and in responsibility) take in this selling. It's a big chance to show what they can do. We have been very happy this fall because we have handled over $1,000 in ticket money alone, which, with the exception of one boy giving out two tickets of the same kind to one girl, has checked to the penny. There seem to be few more palatable in-

centives to good workmanship than a feeling of pleasure in correct results and making money check is one phase of this pleasure.

Of course, especially in these times, there is always the question of indigents, and we have tried to regulate that in these ways. One person in the class checks each morning with the faculty adviser for indigents as to how many tickets have been issued to each indigent and as to whether or not the tickets are loans or are to be earned. Each advisory salesman checks in turn with this person so their records of tickets issued will be constantly correct.

One person in the class is made the foreman of the pupils who are working for tickets. He checks their time and their jobs and pays them in scrip, at the rate of 25¢ an hour, until they have worked off their amount. This foreman is assisted by a committee from the Student Council in his assigning of jobs but he keeps all the records and makes all payments. Here is another place where our mathematics integrates with other school activities.

This ticket sale takes the major part of our time the first month of school as it is supplemented by lessons on business forms, business etiquette, and filing because if our filing is incorrectly done we are not able to easily look up our ticket records. We also do filing for the office for additional practice. At the end of the semester we are also responsible for refunds. The Practical Arts tickets are the type of the usual meal tickets where amounts are punched off as used. Caring for these non-punched tickets and refunding money at the end of the year also makes a good problem.

Our Michigan Theater also cooperates with us in allowing us to attach three movie tickets to our student activity ticket. These admit students to the Saturday morning movies and a representative of the mathematics class must check with the theater at stated periods, figure out the money due, and pay the bill.

As you will notice, there are a great number of individually assigned tasks in this—as we are handling all the money which goes through the school.

Our lunchroom cashier is a general mathematics student (after the first month is over and the ticket sales are out of the way) and each day's lunchroom money is checked by a member of the mathematics class. Adding correctly the prices of various foods on a luncheon plate isn't easy and we have to be careful who is selected for this position. This is an incentive for learning how to add well. Everyone who takes this cashier's position seems to enjoy it and all are anxious to try it. There are also pupils who work in the lunchroom—washing dishes and tables and sweeping floors. These people are usually indigents who are given their lunches and their time is checked and they are paid in scrip by a paymaster. Both the foreman of the ticket earning squad and the lunchroom paymaster must keep a careful check on the scrip he has issued and the scrip he is holding to be sure none is lost.

Regarding the lunchroom, we spend also a day or two each semester figuring calories consumed per day by each class member and proper selection of school lunches. The girls in the class usually have some knowledge of this through their home economics work and bring us reference books containing tables whereby we can count out calories and discuss balanced rations, for the boys this is usually a new and interesting field.

We also welcome any problems wherein price estimates on amounts of food for a picnic or party are necessary and proportionate individual assessments have to be made.

At the beginning of each semester we have been selling tabs and dividers which are used in student notebooks. This necessitates the salesman investigating prices in four or five bookstores; figuring on quality as well as price; deciding on necessary per cent of profit and selling price;

estimating the number to be ordered; reporting to me and, by letter, to the principal; placing the order and paying the bills.

There are, too, one or two members of the class in charge of auditorium activities which require tickets. It is their business to decide upon the approximate number of tickets needed, cooperate with the auditorium committee on the plan of selling, see that tickets are bunched and issued and make final check on money and tickets returned. They must also keep their ticket supply intact. In some cases we have taken over the advertising in the school paper and a little quick poster-making as an additional experience. We are usually very busy but when the control of advertising is necessary for correlation in the mathematics class we take it over.

Last semester we supplemented our school service activities by using a set of drill books in addition, subtraction, multiplication and division; a workbook in elements of business training and a textbook in ninth grade mathematics. In order to do this it was necessary that each person pay an amount equivalent to the price of a new textbook. Our first project in class therefore was to figure out how many of each kind of book we would need for the best results; what books each person was to buy; place the orders and see that the books were distributed and paid for. Each person in the class presented a plan for purchase and we talked them over and chose the one we thought best. Everyone then wrote a letter ordering these books and we again attempted to choose the most businesslike letters and send them. This worked in well at this time for they had just been writing business letters in English. Nothing was said previous to this letter writing about the manner in which the books were to be paid for but plenty of discussion arose later. Some wrote "please send the books," giving price and number desired but made no mention of payment. Others said—"We are sending $20 to pay for these books," etc. So we had a very nice chance to discuss different methods of sending money and actually use the ones we chose. Parcel post rates also came in for discussion here. This semester we have not found it necessary to do this again, as second hand books are available, but we will be placing other orders and will develop these points through them. For example, movies are shipped to us, either by express or parcel post, the parcel post being cheaper, but necessitating our taking them to and from the post office and we can compare methods of shipping here.

As concerns school budgeting, we haven't the background for doing much of it as yet. Last semester we worked out carefully a report as to how the 50¢ fee which each child pays into the social studies fund was spent, using percentages and graphing our results. They were all very interested and from those figures we may be able to help develop the social studies budget for this year.

This mention of the social studies fund brings to my mind the subject of our school truck. Our children have purchased a 210 acre camping site about 200 miles from Ann Arbor where they may go on weekend trips. The camp is being built up gradually with pupil participation and will provide us more and more mathematical material as it develops. To afford transportation to this camp and for our weekly nearby excursions which some school class takes every Wednesday afternoon, we purchased a secondhand truck. This contrivance has caused a great deal of amusement and distress because of its tendency to break down—but has been a great boon in practical experiences to the service classes in Practical Arts and has been a beautiful starting point for discussion of installment plan buying, cost of operation of cars, and secondhand buying in general. On the other hand, we can bring in long-time service and foresightedness in buying to show that even though the bills on the truck have been

plentiful that before it entirely disintegrates it will have saved us considerable money in bus chartering and allowed us to make educational excursions which could not otherwise have been afforded.

At present our class representative has made out with the help of the class and the excursion committee a record form for keeping track of the income of each trip from individual pupil assessments, the cost of each trip for food, repairs, equipment bought or building materials used. He has also made out an expense account form for all excursions and will aid the faculty excursion chairman in reporting on these trips and keeping the records.

The class has formerly computed homeroom scholarship averages and attendance and tardiness percentages for homerooms each six weeks. In order that mistakes in computation may be guarded against these charts are rotated and each person does two or three independently.

Making age and weight graphs of various classes for the school doctor; helping party decorating committees with estimates on cost and amounts of crepe paper, wire, etc., which is needed; helping refreshment committees keep within their budget; buying and selling pop, icecream and candy in concession stands for school events and keeping good understandable record of expense for future use; estimating the proper length of a rope for the flag-pole when the old rope mysteriously disappeared on Hallowe'en, and even drawing to scale a plan for a rat maze for a boy to make in practical arts for a home economics experiment has come into our sphere of activity.

These last mentioned are incidental but, seriously, they go a long way toward making such things as scale drawing, figuring cost and selling prices and per cent of profit and loss more interesting.

The discussion has been limited to the location and utilization of social mathematics in school activities only, but in order that you may know that we have used other social activities also it is perhaps permissable to just mention a few relevant problems. Last semester we investigated our parents' professions or businesses and we had some very interesting reports and problems based on the mathematics used in various occupations. This semester we have established a payroll. Each person in the class makes out a daily time sheet and figures on the basis of 60¢ an hour, the amount he has earned each week for time spent in these mathematical activities. These are handed to me with notes explaining how any amount of time over $1\frac{1}{2}$ hours per day is used. This gives me an interesting insight into the ways different pupils work and a revealing report of their study habits. They are checked by paymasters and the pupils are paid by dummy checks. Each pupil plans the spending of his or her check and through a discussion of certain selected accounts we can motivate our study of savings, investments, interest, insurance, budgeting, wages, and cost of living—I might also mention that this wage of 60¢ was arrived at by the class after investigation and discussion and perhaps a little inflation. In my executive capacity I cut wages if the work is not done properly, and the next week's wage is figured on the new basis.

It's rather difficult to discuss all the material used without making it sound too involved. Careful organization is the most necessary feature from the teacher's standpoint. For the pupil the thing that must come first is a feeling of responsibility. If he fails, more than once, to do a task, he almost invariably interferes with the work of the whole class and after the class has had to do its own work plus the work of one of its delinquent members a few times the social censure does help to develop that needed dependability. Promptness is also a necessary virtue if class work is to be efficient. Each person has an independent job to do as a part of a whole scheme of work and if he fails he disturbs the whole set-up.

This discussion recounts, as adequately

as possible, a few instances wherein the mathematics present in school activities has been used as a basis for a ninth grade class course of study where it seems peculiarly adaptable to the needs of the pupils enrolled and of the school of which they are members.

Of course, these problems would be different in every school set-up, perhaps with only a few fundamentals identical and there are no doubt many, many more ways in which we can use the mathematical facilities of our school organization which have not been utilized. There is no desire to make any unfounded claim and there are no objective results on the effect of this work on these individuals. Each profits by that which he really experiences and each seems interested in the work, is pleased with the responsibility and the trust placed in him, and really becomes quite conscious of the important part mathematics plays in his own world outside of the classroom.

◆ THE ART OF TEACHING ◆

A NEW DEPARTMENT

A Device to Help Beginners in Geometry

By MARGARET AMIG, *Roosevelt High School, Washington, D.C.*

I HAVE found a very slight departure from the usual arrangement of a proposition in plane geometry, very effective in helping pupils to surmount a common difficulty and to avoid a common error. Most beginners find it hard to see why formal proofs of geometric facts are necessary and some are openly rebellious at the idea of giving tedious demonstrations of the truth of very obvious conclusions. Furthermore many pupils approach formal geometry with a background of intuitive geometry and facts retained from that study are likely to increase the pupil's reluctance to substitute reasoning for inspection. The accepted arrangement of a proposition—first theorem, then figure, then proof—adds, it seems to me, to his confusion. Why, he asks, should one write at the top of the page or recite that the base angles of an isosceles triangle are equal and then proceed to prove that it is true?

Closely connected with this common difficulty of appreciating the method of geometry is the common error of quoting as a reason, the theorem which is being proved. Because he has been convinced by inspection from the beginning of the proof that the conclusion is true, the pupil does not hestitate to use it. It seems to be especially satisfying to him to employ it as a last reason because it contains the hypothesis on which the whole demonstration rests. As a final reason it seems to sum up and round out the whole argument.

To help pupils develop good habits of reasoning with suspended judgment, I teach them for the first few months to write the theorem to be proved, at the bottom of the page instead of at the top, and to recite it at the end of the proof rather than at the beginning. This arrangement of a proposition—figure, proof, theorem—centers the pupil's attention on what is new and interesting: namely, the process of reasoning from hypothesis to conclusion, rather than on the discovery of facts, which are probably both stale and self-evident to him. Since the theorem is written after the proof is complete there is no temptation to use it as a reason; at the same time it is available for succeeding proofs and the pupil begins to appreciate the logical development of the series of propositions. During a review of proofs that have been learned, the use of the theorem to indicate which one is to be written or recited upon will accustom the pupil to hearing it before the demonstration and very soon he will be following the same arrangement as the text, without realizing its "formality" or strangeness.

NATIONAL COUNCIL MEMBERS

Be sure to vote by using the Official Primary Ballot on the next page if you wish to be represented in the work of the Council!

on the next page

Official Notice

As secretary of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of the National Council, said election to take place at Chicago, Illinois, on Saturday, February 20, 1937. Article III Section 7, of the by-laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity through announcement through the official journal to suggest by mail for the guidance of the directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the secretary of the board of directors shall send to each member an official ballot giving the names of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the board of which the secretary shall be chairman. The election shall be by mail or in person and shall close on the date of the annual meeting."

At the Detroit meeting, 1931, of the National Council, the nominating committee consisting of the two most recent ex-presidents and the secretary as chairman (for this year: William Betz, J. O. Hassler, and Edwin W. Schreiber), was instructed to prepare a primary ballot suggesting five eligible candidates for each elective office. The officers to be elected at the Chicago meeting are: Second Vice-President, 1937–39 and three directors, 1937–40.

The periods of service of the officers of the National Council, from its organization in 1920 to the present time, are given on page 348.

Edwin W. Schreiber, *Secretary*

Official Primary Ballot

The National Council of Teachers of Mathematics
Chicago Meeting, February 19–20, 1937

For Second Vice-President, 1937–39 (Vote for Two)

( ) Jablonower, Joseph New York, New York	( ) Johnson, John T. Chicago, Illinos	( ) Kearney, Dora Cedar Falls, Iowa
( ) Kee, Olive A. Boston, Massachusetts	( ) Morton, Robert L. Athens, Ohio	

For Members of the Board of Directors, 1937–40 (Vote for Six)

( ) Betz, William Rochester, New York	( ) Hoel, Lesta Portland, Oregon	( ) Shoesmith, Beulah Chicago, Illinois
( ) Butler, Charles H. Columbia, Missouri	( ) Holroyd, Ina E. Manhattan, Kansas	( ) Shuttlesworth, Jos. Summit, N. J.
( ) Christofferson, H. C. Oxford, Ohio	( ) Reese, Dora Grand Rapids, Mich.	( ) Wheeler, Dorothy Hartford, Conn.
( ) Darnell, Alice Philadelphia, Pa.	( ) Roche, Nanette Baltimore, Maryland	( ) Whitcraft, L. H. Muncie, Indiana
( ) Dearman, Dewey S. Hattiesburg, Miss.	( ) Seidlin, Joseph Alfred, New York	( ) Woolsey, Edith Minneapolis, Minn.

Members will please mark this ballot and mail same to Edwin W. Schreiber, Secretary, 719 West Adams Street, Macomb, Illinois. Kindly place name and address on outside of envelope. If you prefer to make a copy of this ballot on a separate sheet of paper it will be acceptable.

The National Council of Teachers of Mathematics

Organized 1920—Incorporated 1928

Periods of Service of the Officers of the National Council

Honorary Presidents

H. E. Slaught, Chicago, Ill., 1936

Presidents

C. M. Austin, Oak Park, Ill., 1920
J. H. Minnick, Philadelphia, Pa., 1921–1923
Raleigh Schorling, Ann Arbor, Mich., 1924–1925
Marie Gugle, Columbus, Ohio, 1926–1927
Harry C. Barber, Exeter, N. H., 1928–1929
John P. Everett, Kalamazoo, Mich., 1930–1931
William Betz, Rochester, N. Y., 1932–1933
J. O. Hassler, Norman, Okla., 1934–1935
Martha Hildebrandt, Maywood, Ill., 1936–1937

Vice-Presidents

H. O. Rugg, New York City, 1920
E. H. Taylor, Charleston, Ill., 1921
Eula Weeks, St. Louis, Mo., 1922
Mabel Sykes, Chicago, Ill., 1923
Florence Bixby, Milwaukee, Wis., 1924
Winnie Daley, New Orleans, La., 1925
W. W. Hart, Madison, Wis., 1926
C. M. Austin, Oak Park, Ill., 1927–1928
Mary S. Sabin, Denver, Colo., 1928–1929
Hallie S. Poole, Buffalo, N. Y., 1929–1930
W. S. Schlauch, New York City, 1930–1931
Martha Hildebrandt, Maywood, Ill., 1931–1932
Mary A. Potter, Racine, Wis., 1932–1933
Ralph Beatley, Cambridge, Mass., 1933–1934
Allan R. Congdon, Lincoln, Nebr., 1934–1935
Florence Brooks Miller, Shaker Heights, Ohio, 1935–1936
Mary Kelly, Wichita, Kansas, 1936–1937

Secretary-Treasurers

J. A. Foberg, Chicago, Ill., 1920–1922, 1923–1926, 1927, 1928 (Appointed by Board of Directors)
Edwin W. Schreiber, Ann Arbor, Mich., and Macomb, Ill., 1929– (Appointed by the Board of Directors)

Committee on Official Journal

John R. Clark, Editor, 1921–1928
W. D. Reeve, Editor, 1928–
Vera Sanford, 1929–
H. E. Slaught, 1928–1935
W. S. Schlauch, 1936–

Directors

Marie Gugle, Columbus, Ohio, 1920–1922, 1922, 1928–1930, 1931–1933
Johathan Rorer, Philadelphia, Pa., 1920–1922
Harry Wheeler, Worcester, Mass., 1920–1921
W. A. Austin, Fresno, Cal., 1920–1921
W. D. Reeve, Minneapolis, Minn., 1920, 1926–1927
W. D. Beck, Iowa City, Iowa, 1920
Orpha Worden, Detroit, Mich., 1921–1923, 1924–1927
C. M. Austin, Oak Park, Ill., 1921–1923, 1924–1927, 1930–1932
Gertrude Allen, Oakland, Cal., 1922–1924
W. W. Rankin, Durham, N. C., 1922–1924
Eula Weeks, St. Louis, Mo., 1923–1925
W. C. Eells, Walla Walla, Wash., 1923–1925
*Harry English, Washington, D. C., 1925–1927, 1928–1930
Harry C. Barber, Boston, Mass., 1925–1927, 1930–1932, 1933–1935
*Frank C. Touton, Los Angeles, Cal., 1926–1928
Vera Sanford, New York City, 1927–1928
William Betz, Rochester, N. Y., 1927–1929, 1930–1931, 1934–1936
Walter F. Downey, Boston, Mass., 1928–1929
Edwin W. Schreiber, Ann Arbor, Mich., 1928–1929
Elizabeth Dice, Dallas, Tex., 1928, 1929–1931
J. O. Hassler, Norman, Okla., 1928, 1929–1931, 1933
John R. Clark, New York City, 1929–1931
Mary S. Sabin, Denver, Colo., 1929–1930, 1931–1933
J. A. Foberg, California, Pa., 1929
C. Louis Thiele, Detroit, Mich., 1931–1933
Mary Kelly, Wichita, Kan., 1932
John P. Everett, Kalamazoo, Mich., 1932–1934
Elsie P. Johnson, Oak Park, Ill., 1932–1934
Raleigh Schorling, Ann Arbor, Mich., 1932–1934
W. S. Schlauch, New York City, 1933–1935
H. C. Christofferson, Oxford, Ohio, 1934–1936
Edith Woolsey, Minneapolis, Minn., 1934–1936
Martha Hildebrandt, Maywood, Ill., 1934–1935
M. L. Hartung, Madison, Wis., 1935–1937
Mary A. Potter, Racine, Wis., 1935–1937
Rolland R. Smith, Springfield, Mass., 1935–1937
E. R. Breslich, Chicago, Ill., 1936–1938
L. D. Haertter, Clayton, Mo., 1936–1938
Virgil S. Mallory, Montclair, N. J., 1936–1938

* Deceased

How Mathematics Associations May Affiliate with The National Council of Teachers of Mathematics

THE National Council of Teachers of Mathematics has many branches in the different states. The aims of the National Council are:

1. To create and maintain interest in the teaching of mathematics.
2. To keep the values of mathematics before the educational world.
3. To help the inexperienced teacher to become a good teacher.
4. To help teachers in service to become better teachers.
5. To raise the general level of instruction in mathematics.

Many other state associations and local clubs have expressed a desire to affiliate with the National Council.

To do this an Association that wishes to affiliate merely passes a resolution at a regular meeting stating that it wishes to affiliate with the National Council, next the blank given below is filled out and forwarded to the Secretary of the National Council, Edwin W. Schreiber, Macomb, Ill. The Secretary will then issue a certificate of affiliation.

The National Council of Teachers of Mathematics
Edwin W. Schreiber, Secretary-Treasurer
State Teachers College, Macomb, Illinois

Date ....

1. Name of organization ....
   .... City .... State ....
2. Time and place of meeting ....
   ....
3. Brief statement of history—when organized, etc. ....
   ....
   ....
   ....
4. How many members .... 5. How many belong to the National Council ....
6. List of Officers for the year 19....
   Name .... Address ....
   Name .... Address ....
   Name .... Address ....
   Name .... Address ....
7. When was resolution passed stating that the organization desires to become affiliated with the National Council ....
8. Who has been appointed as local editor for the *Mathematics Teacher*, to send in newsnotes to the journal, etc.
   Name .... Address ....
9. Suggestions ....
   ....
   ....

(Signed) ....

If available please send complete list of members, using separate sheet.

The new branch of the National Council will then notify the Second Vice-President, Mary Kelly, High School—East, Wichita, Kansas who keeps a record of the name and officers of the affiliated organizations, and the activities of the different branches. Any further inquiries may be addressed to the Second Vice-President.

"In unity there is strength." May we have a number of new branches of the National Council early this year!

MARY KELLY

Importance of Mathematics

IF WE are not greatly mistaken, mathematics is presently destined to play a much greater part in our general scheme of education than it ever has in the past. One is forced to this conclusion not by the insistent demand of students, but by the consideration that the tools and methods offered by this science have been so largely responsible for the extraordinary advances in other sciences which the past generation has witnessed.

The Saturday Evening Post

IN OTHER PERIODICALS

By Nathan Lazar
Alexander Hamilton High School, Brooklyn, N.Y.

Algebra

1. Geil, Johanna. *Coöperative class work in ninth grade algebra.* School Science and Mathematics. 36: 693–46. June 1936.

A description of an interesting experiment to increase the amount of learning obtained from the interaction between pupil and pupil rather than from that between pupil and teacher.

2. Karpinski, L. C. *Simultaneous quadratics solvable in quadratic irrationalities.* American Mathematical Monthly. 43: 362–66. June–July 1936.

The author discusses, from an elementary point of view, the conditions under which the solution of a pair of simultaneous quadratic equations will reduce to the solution of quadratics. He claims, moreover, that "the fundamental question of the types of simultaneous quadratics which are solvable in terms of quadratic irrationalities is, in general, quite inadequately discussed."

3. Paley, George L. *A unit of statistics in ninth year mathematics: an experiment.* High Points. September 1936, pp. 16–25. Vol. 18, no. 7.

A description of a well-planned unit in statistics of four weeks' duration, that was tried out in the Lew Wallace Junior High School, in Brooklyn, New York. The article discusses the following topics:

1. The nature of the group studying the unit.
2. The scope of the course.
3. The final examination given the group.
4. An analysis of the results of the final test.
5. The reaction of pupils to the unit.

On the basis of a questionnaire the writer arrives at the following conclusions:

1. "The overwhelming majority of the pupils in this group evinced a keen interest in the subject of statistics.
2. "Most of them did not find the course difficult.
3. "The class was fully aware of the need for the study of statistics.
4. "Almost all of them believed that statistics has a place in the high school mathematics course of studies.
5. "The class enjoyed the unit.
6. " . . . On the basis of the analysis of the work done on the final examination in the course, it is safe to state that the unit was a huge success with the group to whom it was given.
7. " . . . If this group is a fairly representative sample of the high school population of ninth year pupils, then the introduction of a unit of statistics in connection with the course in elementary algebra can be considered favorably."

4. Yanosik, George A. *A graphical solution for the complex roots of a cubic.* National Mathematics Magazine. 10: 139–40. January 1936.

The writer states a theorem with the help of which he is able to obtain from a graph of a cubic equation its complex roots.

Geometry

1. Bussey, W. H. *Geometric constructions without the classical restriction to ruler and compasses.* The American Mathematical Monthly. 43: 265–80. May 1936.

A very informative treatment on a topic of perennial interest to teachers and students of elementary geometry. By the phrase "geometric constructions without the classical restrictions" the author does not have in mind complicated instruments for drawing, such as the conchoid, quadratrix, etc., but rather such familiar and simple instruments as the marked ruler, parallel ruler, draftman's triangles, T-square, and the carpenter's square.

The examples and illustrations are numerous, and bibliographical references are given throughout the article.

2. Dickter, M. Richard. *The introduction to plane geometry.* School Science and Mathematics. 36: 585–91. June 1936.

The author points out the various methods of introducing geometry, and discusses the advantages and disadvantages of each approach. He then presents an introduction to geometry that he has worked out, summarizes the 15 essential features of his plan, and gives a résumé of the approximate ground to be covered in each of the first twenty lessons, which include three congruence theorems and the theorem on

the equality of the base angles of an isosceles triangle.

3. Hummer, Vivian L. *A comparison of I.Q. and achievement in plane geometry.* School Science and Mathematics. 36: 496–501. May 1936.

The purpose of the study is to compare intelligence quotients and achievement in plane geometry. The specific questions that the author attempts to answer are the following:

a. "Is there a significant correlation between scores on intelligence tests and on objective geometry tests?

b. "Are levels of ability in geometry bounded by specific I.Q. levels?"

After describing in great detail the technique used in arriving at the answers to the above questions, the writer concludes in part, as follows:

a. "There is a significant positive correlation (.58) between scores on the Otis Group Intelligence Examination and on the Columbia Research Plane Geometry Test . . .

b. " . . . It appears that achievement in geometry can be differentiated only at the extremes of intellectual ability.

c. "The wide range of score in each quartile makes it seem inadvisable to attempt to set specific I.Q. levels corresponding to all possible degrees of success in geometry. The evidence does indicate, however, that failure is likely to occur if the I.Q. is below a limit lying somewhere between 100 and 110. The broadening of our basis of prediction to include other factors beside intelligence would probably make a more definite prediction possible."

4. *Rider-work in geometry.* The Mathematical Gazette. 20: 93–109. May 1936.

A report of a discussion held at the annual meeting of *The Mathematical Association* (England), on January 3rd, 1936. The report contains many illuminating and provocative comments on one of the most important and difficult phases of the teaching of geometry—the solution of originals.

Miscellaneous

1. Bedford, Fred L. *Planning the mathematics classroom.* The School *Executive.* April 1936. Pp. 290–92.

With the help of architects' sketches and drawings the author describes a way of transforming the bare and uninteresting mathematics classroom into a fascinating and attractive *mathematics laboratory.* A detailed list of suggested equipment includes the following items: chart files, shelves and table drawers, teacher's file, museum case and show case, models, general equipment, instruments, bulletin board, and blackboard tools. Wherever possible the name of the manufacturer of each of the suggested items is given.

2. Blake, Sue Avis. *Some of nature's curves. II. Simple periodic motion.* School Science and Mathematics. 36: 486–89. May 1936.

The second of a series of articles on the occurrence of familiar curves in natural phenomena.

3. *Exhibition of mathematical films.* The Mathematical Gazette. 20: 110–14. May 1936.

A description of the mathematical films exhibited at the meeting of *The Mathematical Association* (England), on January 3rd, 1936, and a report of the discussion that followed. The interest shown in the demonstration of the films was so great that the General Teaching Committee has appointed a film sub-committee. "Its duties are to watch the production of mathematical films and to collect such information as may make it possible to stimulate and guide that production so that it may be of most use to teachers."

Seemingly our brethren across the sea are stealing a march on us in the field of educational motion pictures in the technical phase of which our country is presumably far more advanced.

4. George, J. S. *In praise of mathematics.* School Science and Mathematics. 36: 509. May 1936.

Some pithy comments on the disappointment that the beginner in mathematics often feels. The author points out the reason for this common experience and enumerates the ultimate achievements accruing to one who is in the possession of the fundamental ideas of mathematics.

5. Johnson, Alwin W. *Trends in high school mathematics.* School Science and Mathematics. 36: 468–70. May 1936.

A statistical study of the place of mathematics in the Nebraska Secondary Schools, during the years 1899–1935. The percentage of schools offering the various mathematical subjects, and the amount of time devoted to each are presented by means of graphs and tables.

6. Weaver, J. H. *As to teaching procedures.* National Mathematics Magazine. 10: 141–42. January 1936.

The first of a series of articles on general or specific procedures in the teaching of elementary college mathematics.

7. *Work for university entrance scholarships.* The Mathematical Gazette. 20: 73–87. May 1936.

A report of a discussion held at the annual meeting of *The Mathematical Association* (England) on January 3rd, 1936.

◆ NEWS NOTES ◆

The Wichita Mathematics Association, a new local branch of the National Council of Teachers of Mathematics, held its first meeting of the year October 5th from four to five-thirty at High School—East.

The officers of this association are: President—Prof. Arthur J. Hoare, University of Wichita; Vice President—Prof. C. A. Reagan, Friends University; Secretary—Miss Ruth Woodworth, High School—East. Treasurer—Mrs. Edith Webster, Hamilton Intermediate School. Local Editor—Miss Mary Kelly, High School—East.

The Certificate of Affiliation has just been received from the Secretary of the National Council, Edwin W. Schreiber.

This was guest day. The purpose of the meeting was to greet the members who have just returned from their vacation, to welcome new members and friends of the association. It was a most enthusiastic meeting, there were about seventy-five present. Plans for the ensuing year were discussed.

Our guest speaker for the afternoon was Dr. A. M. Harding of the University of Arkansas. Dr. Harding talked informally on the New Proposed World Calendar. This was a most interesting and instructive talk. The next meeting will be held late in November at the University of Wichita.

MARY KELLY, *Local Editor*

A joint meeting of the Range Mathematics and Science Clubs was held in Eveleth, Park Hotel, on April 30, 1936. The Range Superintendents were invited to attend this meeting. This meeting was attended by fifty-two mathematics, forty-eight science teachers, and six superintendents of the various school districts represented by the organizations.

The program consisted of the following numbers:

1. Community singing directed by Mr. L. M. Jacobson.
2. Address of welcome by Dr. A. D. Gillette, Supt. Eveleth Public Schools.
3. Vocal solo by Miss Isle Farley accompanied by Miss Florence Nyland.
4. A scientific and mathematical demonstration on the constellations and planets by Mr. Theodore Soloski, Eveleth.
5. Address "Development of Certain Social Aspects in Education" by Dr. M. E. Haggerty Dean, College of Education, University of Minnesota.

Miss M. Robinson, Mr. P. B. Tuttle, and Mr. W. A. Porter were in charge of general arrangements.

In a business meeting of the mathematics teachers the following officers were elected for the ensuing year: H. G. Tiedeman, Mt. Iron, President; P. B. Tuttle, Eveleth, Vice President; Miss Evelyn Hoke, Chisholm, Secretary and Treasurer.

The first meeting of next year will be held at Chisholm with Miss Evelyn Hoke and Mr. W. A. Porter in charge of general arrangements.

A meeting of the Executive Committee for the Range Mathematics Club was held September 21, 1936 at Mountain Iron, Minnesota. A tentative schedule of the meetings for the school year was drawn up. It was decided that the first meeting would be held in Chisholm during the second week in October; the second meeting at Mountain Iron during the first week in December; the third at Aurora during the second week in February; and the fourth and last meeting for the year to be held in Gilbert during the second week in April.

The Range Science Club, with Mr. W. A. Porter, Chisholm, as president, also meets at the same time and places as the Mathematics Club. At Chisholm the two clubs will meet jointly with Dr. Piepkorn, Chisholm, as their principal speaker. Miss Evelyn Hoke, Chisholm, and Mr. W. A. Porter are in charge of arrangements.

The Range Executive Committee consists of the representatives listed below:

1. Aurora—Miss Josephine Sharp, Senior High School
2. Biwabik—Miss Anna Johnson, Senior High School
3. Buhl—Mr. Kevin Keenan, Senior High School
4. Chisholm—Miss Evelyn Hoke, O'Neil Hotel
5. Ely—Miss E. Luella Holt, Senior High School
6. Eveleth—Mr. P. B. Tuttle, Junior High School
7. Gilbert—Mr. Ole Shey, Senior High School
8. Hibbing—Mr. L. M. Becker, Junior College
9. Leoneth—Mr. L. Gerdee, Senior High School
10. Mt. Iron—Mr. H. G. Tiedeman, Senior High School
11. Tower—Mr. Ralph Iverson, Senior High School

12. Virginia—Miss Mina B. Johnson, Junior High School

H. G. TIEDEMAN, *President*
Range Mathematics Club

The meeting of the Mathematics Section of the California Teachers Association, Bay Section, was held at the International House in Berkeley on May 16, 1936. The following were the guests at a luncheon which preceded the business meeting: Aubrey A. Douglass, Chief of the Division of Secondary Education, State Department of Education; Dr. G. C. Evans, Chairman of the Mathematics Department at the University of California; B. A. Bernstein, Professor of Mathematics, University of California; E. C. Goldsworthy, Assistant Professor of Mathematics and Assistant Dean of Undergraduates, University of California; Miss S. H. Levy, Assistant Professor of Mathematics, University of California; T. M. Putnam, Professor of Mathematics and Dean of Undergraduates, University of California; Harold M. Bacon of Stanford University.

Dr. Douglas was particularly anxious for the mathematics teachers to work out a reorganization of their courses. He is certain that the subject is fast losing ground and that it needs revitalizing. The university instructors offered to cooperate with the mathematics teachers working on the problem. As a result, it was decided that two committees were to be selected, one to work on the reorganization of mathematics for a college preparatory course; the other, for a course to meet the everyday life situation involving mathematics.

The Nominating Committee reported and their choice of officers was unanimously elected. They are as follows: Advisory Chairman: Mrs. Helen Hoefer, Richmond High School. Chairman of West Bay: Mary McBride, Supervisor of Mathematics and Science in the Junior High Schools, San Francisco. Secretary of West Bay: Adeline Scandrett, Presidio Junior High School, San Francisco. Chairman of East Bay: Dean G. Smith, Bret Harte Junior High School, Oakland. Secretary-Treasurer of East Bay: Bernice Cochran, Fremont High School, Oakland.

The motion was made and seconded that some kind of Seminar Course be worked out at the University of California to enable the teachers to exchange ideas and work out these two projects.

RUTH PETERSEN, *Secretary-Treasurer*
Mathematics Section for 1935–36

The Minneapolis Mathematics Club

The Minneapolis Mathematics Club completed its active program for the year 1935–36 under the leadership of its president, Dr. L. B. Kinney, head of the mathematics department of the University of Minnesota High School. Six varied programs included the following stimulating talks:

1. "The Course of Study Now in Preparation for the Junior High Schools of Minneapolis" by C. E. Reichard, principal of Jefferson Junior High.
2. "The Residue of High School Learning" by Dr. A. C. Eurich, assistant to President Coffman of the University of Minnesota.
3. "Why Teach Geometry" by C. O. Bemis, head of the mathematics department at the St. Cloud State Teachers College.
4. "Consumer Mathematics" by Dr. L. B. Kinney of the University High School.
5. "The Unified Curriculum" by Dr. O. R. Floyd, principal of University High School.
6. "Mathematics in the Schools of Germany" by H. J. Burton, an instructor in the high school at Brainerd, Minn.

Two members of the club, Edith Woolsey and Laura Farnam, who attended the Dec. 31–Jan. 1 meeting in St. Louis of the National Council of Mathematics Teachers, also brought back reports of that meeting.

CLARA A. JOHNSON, *Secretary*

The Mathematics section of the State Teachers Association of North Carolina met at Raleigh on March 20th. The following program was given.

Appointment of Committee on Nominations.

1. "How Plane Geometry Can be Reorganized to Fit an Eight Months Term." Ida Belle Moore, Senior High School, Greensboro, N. C.
2. "Bridging the Gap Between the Junior High School and the Senior High School." Eura Strother, Junior High School, Durham, N. C.
3. "A Brief Professional Philosophy for Teachers of Mathematics." H. F. Munch, University of North Carolina, Chapel Hill.

Election of officers and other business: New President—E. F. Canaday of Meredith College, Raleigh, N. C. Local Editor—H. F. Munch.

American Education Week

"Oh," said the Bewildered Young Teacher, "I've heard about American Education Week, of course, but I never did anything about it. The National Education Association, The American Legion, and the United States Office of Education take care of that."

"My Young and Beautiful Friend," said the Wise Old Duck, "Did it ever occur to you that, as far as this community is concerned, you are the National Education Association, your brother-in-law is the American Legion, and the parents of your pupils are the United States Office of Education?"

"No," said the Bewildered Young Teacher, "but I still don't see what I can do about anything as big as American Education Week."

"I thought that might be the difficulty," said the Wise Old Duck, settling himself in his chair and tugging at his sideburns. "You're confusing American Education Week with Chew-More-Chewing-Gum Week. There you have an effort by a few big manufacturers to sell more chewing gum. An advertising man sits down and plans a national campaign. American Education Week, on the other hand, is mainly the sum of what you and several hundred thousand other teachers do to explain our work to the parents of our children. Oh, your local and state and national organizations do what they can, but the six million parents who visited American schools during American Education Week last year, did so because each teacher got ten, or twenty, or thirty parents to visit her particular school to see what was being done there."

"But why do that during one particular week," said the B.Y.T., "Why not spread that out over the year?"

"You've heard about 'Everybody's business being nobody's business,' " said the W.O.D. "Something that everybody's going to do sometime, nobody does no time. Forty or fifty-two American Education Weeks each year would be very nice, except that we would forget about them and have to have a special week to remember American Education Weeks."

"But why that particular week of November ninth to fifteenth," insisted the B.Y.T. "Why can't each school pick its own week?"

"There are two good reasons for that. If each school is left to pick its own week, a great many that intend to do so just won't get around to it. The other reason is one of mass psychology. The week that is picked becomes much more significant to teachers, parents, and the general public if it is a nation-wide movement. The newspapers give it more attention. The public is more interested, and the parents are more likely to visit your school if they know that millions of other parents all over the country are visiting the schools at the same time."

"But I want to go away that week-end, and it's too early in the term for me to have a big display ready."

"This won't make it any more convenient for you, of course, but it may reconcile you to know that this particular week is picked because it contains Armistice Day, and the American Legion is not only one of the strongest supporters of American Education Week, but one of the best friends the schools have.

"As for that matter of a display, I'm not sure that that's the best way of doing something about the particular theme of the week. The theme is 'Our American Schools AT WORK.' If I were a parent, I'd rather see what the school is doing than what it has done."

"I suppose that's true," said the B.Y.T., "but I don't know what to do about it."

"You can get any number of suggestions and all sorts of helpful material from the National Education Association," said the W.O.D., "but to tell you the honest truth, it doesn't make a great deal of difference *what* you do. The important thing is to do *something.* Whatever you feel will best inform the parents of *your* children about the work that *you* are doing is probably the best observance for you. Certainly no one could ask more of you than that."

American Education Week

When—November 9–13, 1936

What—"Our American Schools at Work"
- Monday—The Story of the Schools
- Tuesday—The Changing Curriculum
- Wednesday—New Services to the Community
- Thursday—The Unfinished Business of Education
- Friday—Financing America's Schools
- Saturday—Education for Physical Fitness
- Sunday—Education for Character

Where—Washington—Maine
California—Florida

Who—Every School, Every Teacher, Every Friend of Education in cooperation with
The National Education Association
The United States Office of Education
The American Legion

How—Booklets and suggestions may be obtained by writing to:
The National Education Association
1201–16th Street, N. W.
Washington, D. C.

Why—"A movement that causes six million adults to visit the schools, carries a message concerning the schools to ten million laymen, and calls forth special proclamations

from more than thirty-five governors, is a project in educational interpretation which challenges the attention of the entire profession."

At the annual meeting of the Illinois Section of the Mathematical Association of America, held at Normal, Illinois, May 8 and 9, a committee composed of Dr. W. C. Krathwohl of the Armour Institute of Technology Chairman, Dr. E. H. Taylor of Eastern Illinois State Teachers College, and Dr. H. B. Curtis of Lake Forest College, was appointed to draw up a set of resolutions concerning a National Commission on Mathematics in the Elementary Schools.

These resolutions were adopted by the Illinois Section at the Saturday Session. They are as follows:

"Whereas, the Report of the National Committee on the Reorganization of Mathematics in Secondary Education has been of such great value in improving the curriculum and instruction in secondary mathematics, and

Whereas, there exists a need for a similar report on mathematics in the elementary schools by persons competent to conduct such an investigation,

Therefore, be it resolved that the Illinois Section of the Mathematical Association of America urge the Mathematical Association of America to join with the National Council of Teachers of Mathematics to obtain the appointment of a National Commission on Mathematics in the Elementary Schools and to obtain means for its financial support.

These resolutions were adopted,

(1) to initiate a movement for a National Commission,

(2) because of a realization that it is not sufficient to improve the teaching of mathematics by beginning in the secondary schools, any attempt of this kind should begin in the elementary grades.

(3) because of a realization that if mathematicians do not set their house in order, others, particularly pseudo-educators and pseudo-scientists, will do it for them."

W. C. Krathwohl

Resolutions Adopted by the Men's Mathematics Club of Chicago and Metropolitan Area

Whereas, there is an urgent need for an authoritative study of the curriculum and instruction in mathematics in the elementary schools corresponding to the study in the field of secondary mathematics made by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America.

Therefore, be it resolved that the men's Mathematics Club of Chicago and Metropolitan Area urge an early consummation of cooperative plans, already in process of formulation, between the Mathematical Association of America and the National Council of Teachers of Mathematics in sponsoring a National Commission on Mathematics in the elementary schools.

E. C. Hinkle
J. T. Johnson
Marx Holt

The Central Association of Science and Mathematics Teachers will hold its annual Convention at the Coronado Hotel in St. Louis, Missouri on November 27 and 28.

A cash award of $1000.00 is offered by The Williams & Wilkins Company for the best manuscript on a science subject, presented before July 1, 1937.

Literary prizes are relatively common, but it is not so usual for a publisher to be bidding for science material in this manner.

The publishers put no limitations on the subject-matter or manner of handling and none on eligibility for the award. The MS. must be in English and "of a sort calculated to appeal to the taste of the public at large." The desired length is given as 100,000 words.

While any MS. on a science subject will be considered, it is expected that the author will prove to be a man or woman engaged in a scientific pursuit and who is possessed of the requisite literary skill to interpret science for that portion of the public which reads books.

To assure authenticity, the publishers have enlisted the services of some 25 or 30 "advisers," these being men of science or wide reputation and assured competence. One or more of the advisers will pass upon each MS. from the viewpoint of soundness and accuracy.

The award will lie in the joint discretion of four judges selected with a view to their especial qualification in choosing the sort of book that will appeal. These are:

Dr. Joseph Wheeler, Librarian of the Pratt Library in Baltimore, and chairman of the Book List Committee of the Association for the Advancement of Science; Harry Hansen, reviewer and critic for the New York World Telegram and Harper's Magazine; Dr. Lyman Bryson, Professor of Education of Teachers College, Columbia, and Director of the "Readability Laboratory"; and David Dietz, science editor of the Scripps-Howard newspapers.

Further details concerning the award may be had by addressing the publishers at Mt. Royal and Guilford Avenues, Baltimore, Maryland.

Mr. Joseph Jablonower, Chairman of the Mathematics Committee of the Commission on Secondary School Curriculum, addressed the David Eugene Smith Club on Thursday evening, April 23 in the Grace Dodge Room at Teachers College, Columbia University. He outlined the program of the Thayer commission from the point of view of the material already in the curriculum, needs of adolescents and evaluation of teaching.

He discussed the work of the Science Committee, particularly that concerning the mathematical aspects, which he suggested will become increasingly important in the outlook of the average man.

The chief aims to be realized through learning experiences, as outlined by the committee, are: reflective thinking, willingness to act on the basis of tentative judgment, intelligent self-direction, social maturity, constructive personal and social attitudes, creativeness, appreciation, skills and techniques and integration of behavior.

At the May meeting of the Men's Mathematics Club of Chicago Dr. E. R. Breslich discussed the topic "Some Problems in the Teaching of Mathematics."

The Spring Meeting of the Mathematics Section of the California Teachers Association, Bay Section was a luncheon held at International House, University of California at Berkeley on May 16th. A reception preceded the luncheon honoring the following guests:

Dr. A. A. Douglas, Chief of the Division of Secondary Education, California State Department of Education.

Mr. G. C. Evans, Chairman of the Department of Mathematics, University of California.

Mr. T. M. Putnam, Professor of Mathematics, and Dean of Undergraduates, University of California.

Mr. B. A. Bernstein, Professor of Mathematics, University of California.

Miss Sophia Levy, Assistant Professor of Mathematics, University of California.

Mr. E. C. Goldsworthy, Assistant Professor of Mathematics and Assistant Dean of Undergraduates, University of California.

More than a hundred enthusiastic members attended the meeting. Greetings were read from Mrs. F. Brooks Miller and Professor W. D. Reeve of the National Council of Teachers of Mathematics and THE MATHEMATICS TEACHER.

Dr. Douglas presented the case for Mathematics through the Junior and Senior high schools. He pointed out the urgent need for active interest in protecting the place of Mathematics in the curriculum; not some mathematics as needed but systematic courses.

A lively discussion followed his talk the outcome of which was the appointment of two committees to work on (1) the place of Mathematics in the new core curriculum courses, and (2) the plan for a four year college preparatory course.

Professor Evans and his colleagues offered to provide an Extension course next winter for secondary teachers of this section in order that we may all work together for the betterment of the teaching of Mathematics.

The following officers were elected for 1936–1937:

General Chairman—Mrs. Helen T. Hoefer, Richmond Union H. S., Richmond

Chairman of the West Bay Section—Miss Mary K. McBride, Supervisor of Math. and Science in the Junior High Schools of San Francisco

Secretary of West Bay Section—Miss Adeline Scandrett, Presidio Junior High School, San Francisco

Chairman of the East Bay Section—Miss Dean Smith, Bret Harte Junior High School, Oakland

Secretary of East Bay Section—Miss Bernice Cochrane, Fremont High School, Oakland.

The Spring Meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England was held at the Choate School at Wallingford, Conn., on Saturday, April 25, 1936:

PROGRAM

Morning Session

10:00—Social Gathering.

10:30—Welcome. Rev. George St. John, A.B., A.M., LL.D., Headmaster of the Choate School.

"Possible Disadvantages in the New Requirements." William G. Shute, The Choate School.

"A Spotlight on the Congruence Argument." Carroll G. Ross, The High School, Scarsdale, N.Y.

"The Effect of Modern Science Upon Our Teaching of Mathematics." Mrs. Frank L. Boyden, Deerfield, Academy.

1:00—Luncheon.

Afternoon Session

2:15—Business Meeting.

Election of Officers.

"Teaching Selected Topics in Algebra."
Rolland R. Smith
Classical High School,
Springfield, Mass.
and Columbia University

Officers of the Connecticut Valley Section

Nelson A. Jackson, President, Mount Hermon School

Fred L. Mockler, Vice-President, High School, Holyoke

Miss Helen Wright, Secretary, Northfield Seminary

Miss Beatrice Neale, Treasurer, Bulkeley High School, Hartford

Professor David D. Leib, Director, Connecticut College

Miss Sarah K. Evarts, Director, Bulkeley High School, Hartford

Past Presidents

Harry B. Marsh, '20; Percy F. Smith, '21; M. M. S. Moriarty, '22; Eleanor C. Doak, '23; Joe G. Estill, '24; Joshua I. Tracey, '25; *Lyon L. Norton, '26; *John W. Young, '27; Roland R. Smith, '28; Harriet R. Cobb, '29; Melvin J. Cook, '30; Bancroft H. Brown, '31; Dorothy Wheeler, '32; H. W. Dadourian, '33; David D. Leib, '34.

On October 17 the first meeting and luncheon of the Women's Mathematics Club of Chicago and Vicinity was held at the Medical and Dental Arts Building with more than one hundred members attending.

The guest speaker of the afternoon was Professor Harold Fawcett of the Ohio State University who spoke on Human Values of High School Mathematics." Mr. Fawcett showed that mathematics is the only living science and that through mathematics scientific and dependable thinking is attained. No subject in the curriculum is so great a means of molding the thinking of a child as mathematics.

The officers for the year are:

President—Miss Laura E. Christman, Senn High School. Vice President—Miss Marie Chrisler, Kenosha High School. Secretary—Miss Lenore King, Flower High School. Treasurer—Miss Dorothy Martin, Bloom High School. Editor—Miss Ida D. Fogelson, Bowen High School. Program Committee: Miss Clara D. Murphy, Evanston High School, Miss Nettie K. Courtney, Riverside High School, Miss Neva Anderson, Evanston High School.

Ida D. Fogelson

Dr. G. T. Pugh, Professor of Mathematics and Astronomy, of Winthrop College, died on July 1, 1936. He was a charter member of the Winthrop College Branch of the National Council of Teachers of Mathematics.

* Deceased.

The Education Gazette for June 1, 1936 had the following to say about the tenth yearbook of the National Council of Teachers of Mathematics:

"The Council publishes each year a book dealing with some features of the teaching of Mathematics. The Year Book for 1935 presented some of the most important ideas and proposals in the teaching of Arithmetic. The contents comprise a wide survey of the theory and practice of the subject and include chapters on such topics as:—Psychological Considerations in the Learning and the Teaching of Arithmetic; the Relation of Social Arithmetic to Computational Arithmetic; Opportunities for the Use of Arithmetic in an Activity Programme; Economy in Teaching Arithmetic; the Problem of Transfer in Arithmetic; Types of Drill in Arithmetic; the Mathematical Viewpoint applied to the Teaching of Elementary School Arithmetic and the New Psychology of Learning.

"The writers have applied modern psychology to the teaching of arithmetic and have given an excellent account of present tendencies in the American schools. The spirit of the new movement in the teaching of arithmetic is well expressed in the opening chapter:—'The basic tenet in the proposed instructional reorganization is to make arithmetic less a challenge to the pupil's memory and more a challenge to his intelligence.'

"The book would be very helpful to any student of educational psychology as applied to the teaching of arithmetic and is well worth a place in any educational library."

At the April meeting of the Men's Mathematics Club of Chicago W. E. Block of Lake View High School spoke on "Binary Number, The Basic Number System." The concluding argument for "Binary Numbers" was given by Marx Holt, Principal of the Fisk School.

The Chicago Tribune for April 19, 1936 included the following note:

"In session at Kansas City last Monday the American Chemical society approved a committee report sharply criticizing the nation's high school educational system. The report, read by Ross Aiken Gortner of the University of Minnesota, charged that there are many high school instructors teaching subjects in which they have had no academic work, and

that many more have completed no more than a first year college course in their subjects.

"The report asserted that high school graduates of the last ten years are woefully deficient in mathematical knowledge, with the situation 'tending, if possible, toward a worse condition.'

"A majority of the country's noted college professors are made ineligible to teach high school classes, even if they choose to do so, by state monopolies of the educational 'machinery,' the report added."

The Eighteenth Annual meeting of the National Council of Teachers of Mathematics will be held at the Palmer House in Chicago, Illinois, Feburary 19 and 20, 1937.

Room tariff at the Palmer House is as follows:

Single room with bath, per person from $3.00

Double-bed room with bath, per person from $2.50.

Twin-bedded room with bath, per person from $3.00.

Three persons in a room, per person from $2.50.

Four persons in a room, per person from $2.00.

The Palmer House maintains six public dining rooms where breakfast may be had from 40¢ per person, luncheon from 35¢ and dinner from 85¢. Ample parking space and garage facilities will be available within one block of the hotel.

Send in your reservations now.

Professor R. C. Archibald has recently sent us the following errata in his article on Babylonian Mathematics which appeared in the May, 1936, number of THE MATHEMATICS TEACHER:

p. 213, col. 1, ll. 22–23, *for* "papyrus," *read* "tablet,"

p. 214, col. 2, l. 3, *for* "lengths of its base" *read* "lengths of half its base,"

p. 215, col. 1, l. 7, *for* "frustrum" *read* "frustum,"

p. 215, col. 1, ll. 10–11, *for* "volume of the *frustrum of a cone*, or of a square pyramid, is," *read* "volume of the frustum of a square pyramid is."

p. 215, col. 1, l. 15, *for* "frustrum," *read* "frustum."

The Association of Mathematics Teachers of New Jersey held its Fifty-seventh regular meeting at the State University of New Jersey in New Brunswick on May 2, 1936. The following program was given:

"History of Mathematics," Dr. Martin A. Nordgaard, Upsala College.

"The Present Status of the Ninth Grade Syllabus," Mr. Roscoe P. Conkling, Central High School, Newark.

"Progress Report on the Tenth Grade Syllabus," Mr. Ferdinand Kertes, Perth Amboy High School.

Officers of the Society

Professor Albert E. Meder, President, New Jersey College for Women, New Brunswick, N.J.

Miss Marion Lukens, Vice-President, High School, Camden, N.J.

Andrew S. Hegeman, Secretary-Treasurer, Central High School, Newark, N.J.

Council Members

The Officers and Dean Luther P. Eisenhart, Professor Charles O. Gunther, Professor Richard Morris, Professor Virgil S. Mallory, Harrison E. Webb, Miss Amanda Loughren, Ferdinand Kertes, Miss Catherine B. Read.

Mathematics Meeting

Saturday, November 7, 9:30 A.M.

Room 16, Roosevelt High School

Chairman: MISS NELLE M. COOK, Kansas City

THEME: "WHY WE ARE NOT THROUGH WITH MATHEMATICS"

"Mathematics in Music," C. F. Lebow, Salina, Kansas.

Music: Violin Solo—Frasquita, Miss Roberta Lebow.

"Mathematics in Art," Miss Lillian Bohl, Wyandotte High School, Kansas City, Kansas.

"Mathematics in Surveying," E. A. Beite, University of Wichita, Wichita, Kansas.

"Mathematics in Astronomy," Prof. J. A. C. Shirk, K. S. T. C., Pittsburg, Kansas.

"The Teaching of Arithmetic," Miss Winona Campbell, Concordia, Kansas.

"What the College Teacher Expects of the High School Pupil, with some suggestions for its accomplishment," Miss Ina E. Holroyd, Kansas State College, Manhattan, Kansas.

Election of Chairman for 1937.

Exhibit in charge of Salina Mathematics Teachers.

Copies of tests and special work in different topics of mathematics will be available for teachers in attendance.

Breakfast will be served at the Cafeteria, Roosevelt High School, 8:00 A.M. in charge of Salina Mathematics Teachers.

A Permanent Exhibit of Mathematics in Relation to Other Great Fields of Knowledge is now on display at Teachers College, Columbia University.

The exhibit is mostly on the third floor of the main hall of Teachers College but the exhibit has been enlarged and special features are being shown in the mathematics library on the second floor.

This exhibit has been developed under the direction of Professor W. D. Reeve who is interested in trying to bring out the important correlation between mathematics and other fields of knowledge and life situations.

Particular emphasis is paid to the use of arithmetic in the daily affairs of life like budgeting (family, city, state, and federal) installment buying and the mathematics of finance. Many people in this country would be surprised if they knew that instead of 6% interest which they think they are paying for articles bought on the installment plan they are really paying over 25% or 30% interest. There are charts to show how mathematics is applied to the prevention and treatment of diseases, nursing, home economics, Animal husbandry, eugenics, hygiene, sport, factory management, taxation, science, art, architecture and so on.

The exhibit contains considerable material of interest to the layman. For the teachers of mathematics and his pupils there is display of textbooks, testbooks and workbooks that will be of interest. There are also a large number of cases containing models made by pupils as well as models made for sale by commercial concerns. A large number of mathematical instruments for classroom use in teaching mathematics like simple sextants, transits and the like. Some of these instruments are homemade and others are commercially made. A section of the exhibit is devoted to many famous mathematical problems particularly the three famous problems of antiquity; the trisection of an angle, the squaring of a circle, and the duplication of a cube. Space is also given over to games and other mathematical recreations and plays.

Commenting on the exhibit Dr. Reeve said, "I have been interested in the matter of a permanent exhibit and collection of mathematical materials for twenty-five years as a part of my own teaching technique. But as time went on I began to realize what it might mean to teachers of mathematics all over the country if we could have carefully assembled in one place where it would be constantly available for inspection and study. This led me to play a permanent exhibit at Teachers College where teachers now come with their classes to take advantage of the work we have done and to get ideas for starting exhibits of their own. The Women's Mathematics Club of Chicago have now started a similar exhibit which was recently shown in Mandel's store in Chicago.

"It is rare that the ordinary layman realizes what mathematics has done and is still doing to make our lives more comfortable during the last fifty years. In the complex civilization which we are now entering a knowledge of mathematics will be more necessary and important than ever. Great stress is laid today upon the importance of the social studies. What the people of this country do not realize is that arithmetic is the most important of the social studies. It is possible that most of our ills today economic and otherwise can be traced to a wide spread ignorance all over the land of the fundamental ideas of arithmetic.

"If this exhibit can point out what the most important parts of mathematics are that the well educated citizen ought to know and can help teachers to organize and teach the subject better we shall be glad to have had a part in it."

Announcement of Discussion Luncheon at Annual Meeting

The annual meeting of the National Council of Teachers of Mathematics will be held at the Palmer House, Chicago, on February 19 and 20, 1937. There will be general programs and departmental programs for all teachers of mathematics from the lower grades to the college.

Announcements of details of these programs will appeer in the December and January issue of the Mathematics Teacher. At this time, we are announcing a discussion luncheon to be held on Saturday, February 20, from 12:00 until 2:00 o'clock.

The cost of the luncheon will be one dollar and a quarter per plate. It is the purpose of this luncheon to provide a greater opportunity for the interchange of ideas and for more general discussion than can usually be given in a formal meeting. Therefore, the members of the Board of Directors have kindly consented to act as hosts and discussion leaders, each one at a table for eight—at most ten people. The names of the discussion leaders and the topics they have selected for discussion are listed below.

We are giving you an opportunity to make advance reservations. If you wish to attend this luncheon, please send your name, your first, second and third choice in topics and a check for $1.30 (includes state tax) to Mr. E. W. Schreiber, 719 West Adams Street, Macomb, Illinois. He will send you your ticket and the topic for which your reservation has been made. Reservations will be filled in the order in which they are received.

Florence Brooks Miller: "What can be done to increase the interest of the class room teacher in the National Council of Teachers of Mathematics?"

Mary Kelly: "What should the Senior High School course in Mathematics include?"

Edwin W. Schreiber: "Some of the teaching problems of Plane Geometry."

Wm. D. Reeve: "Mathematics and the integrated program."

Vera Sanford: "A course for Normal School Freshmen which will impress them with the importance of mathematics."

W. S. Schlauch: "What are the requirements for a good examination in mathematics?"

Wm. Betz: "The problem of retardation in grades 7 and 8."

H. C. Christofferson: "Meaningful teaching and measuring results."

Edith Woolsey: "Consumers mathematics for the ninth grade."

M. L. Hartung: "What are some of the promising recent developments or trends in the teaching of geometry?"

Mary A. Potter: "All kinds of pictures we use in teaching mathematics."

Rolland R. Smith: "Introduction to geometry."

E. R. Breslich: "Questions relating to the improvement of teaching mathematics."

Leonard D. Haertter: "What should be the contribution of mathematics to the education of children?"

Virgil S. Mallory: "Providing challenging material for the bright pupil."

Back Numbers Available

The following issues of the *Mathematics Teacher* are still available and may be had from the office of the *Mathematics Teacher*, 525 West 120th Street, New York.

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